CHAPTER 4

HEAT TRANSFER

HEAT transfer is energy transferred because of a temperature difference. Energy moves from a higher-temperature region to a lower-temperature region by one or more of three modes: **conduction**, **radiation**, and **convection**. This chapter presents elementary principles of single-phase heat transfer, with emphasis on HVAC applications. Boiling and condensation are discussed in Chapter 5. More specific information on heat transfer to or from buildings or refrigerated spaces can be found in Chapters 14 to 19, 23, and 27 of this volume and in Chapter 13 of the 2006 *ASHRAE Handbook—Refrigeration*. Physical properties of substances can be found in Chapters 26, 28, 32, and 33 of this volume and in Chapter 9 of the 2006 *ASHRAE Handbook—Refrigeration*. Heat transfer equipment, including evaporators, condensers, heating and cooling coils, furnaces, and radiators, is covered in the 2008 *ASHRAE Handbook—HVAC Systems and Equipment*. For further information on heat transfer, see the Bibliography.

HEAT TRANSFER PROCESSES

Conduction

Consider a wall that is 10 m long, 3 m tall, and 100 mm thick (Figure 1A). One side of the wall is maintained at t_{s1} = 25°C, and the other is kept at t_{s2} = 20°C. Heat transfer occurs at rate q through the wall from the warmer side to the cooler. The heat transfer mode is conduction (the only way energy can be transferred through a solid).

- If t_{s1} is raised from 25 to 30°C while everything else remains the same, q doubles because $t_{s1} - t_{s2}$ doubles.
- If the wall is twice as tall, thus doubling the area A_c of the wall, q doubles.
- If the wall is twice as thick, q is halved.

From these relationships,

$$q \propto \frac{(t_{s1} - t_{s2})A_c}{L}$$

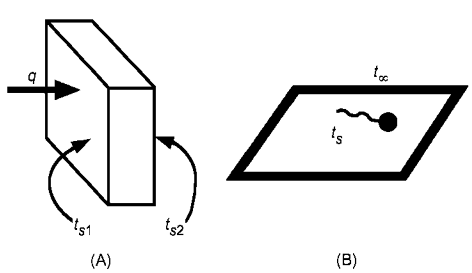


Fig. 1 (A) Conduction and (B) Convection

The preparation of this chapter is assigned to TC 1.3, Heat Transfer and Fluid Flow.

where ∝ means "proportional to" and L = wall thickness. However, this relation does not take wall material into account: if the wall is foam instead of concrete, q would clearly be less. The constant of proportionality is a material property, **thermal conductivity *k***. Thus,

$$q = k\frac{(t_{s1} - t_{s2})A_c}{L} = \frac{(t_{s1} - t_{s2})}{L/(kA_c)} \tag{1}$$

where k has units of W/(m·K). The denominator $L/(kA_c)$ can be considered the **conduction resistance** associated with the driving potential $(t_{s1} - t_{s2})$. This is analogous to current flow through an electrical resistance, $I = (V_1 - V_2)/R$, where $(V_1 - V_2)$ is driving potential, R is electrical resistance, and current I is rate of flow of charge instead of rate of heat transfer q.

Thermal resistance has units K/W. A wall with a resistance of 5 K/W requires $(t_{s1} - t_{s2})$ = 5 K for heat transfer q of 1 W. The thermal/electrical resistance analogy allows tools used to solve electrical circuits to be used for heat transfer problems.

Convection

Consider a surface at temperature t_s in contact with a fluid at t_∞ (Figure 1B). **Newton's law of cooling** expresses the rate of heat transfer from the surface of area A_s as

$$q = h_c A_s (t_s - t_\infty) = \frac{(t_s - t_\infty)}{1/(h_c A_s)} \tag{2}$$

where h_c is the **heat transfer coefficient** (Table 1) and has units of W/(m²·K). The **convection resistance** $1/(h_c A_s)$ has units of K/W.

If $t_\infty > t_s$, heat transfers from the fluid to the surface, and q is written as just $q = h_c A_s (t_\infty - t_s)$. Resistance is the same, but the sign of the temperature difference is reversed.

For heat transfer to be considered convection, fluid in contact with the surface must be in motion; if not, the mode of heat transfer is conduction. If fluid motion is caused by an external force (e.g., fan, pump, wind), it is **forced convection**. If fluid motion results from buoyant forces caused by the surface being warmer or cooler than the fluid, it is **free** (or **natural**) **convection**.

Table 1 Heat Transfer Coefficients by Convection Type

Convection Type	h_c, W/(m²·K)
Free, gases	2 to 25
Free, liquids	10 to 1000
Forced, gases	25 to 250
Forced, liquids	50 to 20 000
Boiling, condensation	2500 to 100 000

Radiation

Matter emits thermal radiation at its surface when its temperature is above absolute zero. This radiation is in the form of photons of varying frequency. These photons leaving the surface need no medium to transport them, unlike conduction and convection (in which heat transfer occurs through matter). The rate of thermal radiant energy emitted by a surface depends on its absolute temperature and its surface characteristics. A surface that absorbs all radiation incident upon it is called a **black surface**, and emits energy at the maximum possible rate at a given temperature. The heat emission from a black surface is given by the **Stefan-Boltzmann law:**

$$q_{emitted,\,black} = A_s \sigma T_s^4$$

where $E_b = \sigma T_s^4$ is the **blackbody emissive power** in W/m^2; T_s is absolute surface temperature, K; and $\sigma = 5.67 \times 10^{-8}$ W/(m$^2 \cdot$K^4) is the Stefan-Boltzmann constant. If a surface is not black, the emission per unit time per unit area is

$$E = \varepsilon \sigma T_s^4$$

where E is emissive power, and ε is emissivity, where $0 \le \varepsilon \le 1$. For a black surface, $\varepsilon = 1$.

Nonblack surfaces do not absorb all incident radiation. The absorbed radiation is

$$q_{absorbed} = \alpha A_s G$$

where **absorptivity** α is the fraction of incident radiation absorbed, and **irradiation *G*** is the rate of radiant energy incident on a surface per unit area of the receiving surface due to emission and reflection from surrounding surfaces. For a black surface, $\alpha = 1$.

A surface's emissivity and absorptivity are often both functions of the wavelength distribution of photons emitted and absorbed, respectively, by the surface. However, in many cases, it is reasonable to assume that both α and ε are independent of wavelength. If so, $\alpha = \varepsilon$ (a **gray surface**).

Two surfaces at different temperatures that can "see" each other can exchange energy through radiation. The net exchange rate depends on the surfaces' (1) relative size, (2) relative orientation and shape, (3) temperatures, and (4) emissivity and absorptivity. However, for a small area A_s in a large enclosure at constant temperature t_{surr}, the irradiation on A_s from the surroundings is the blackbody emissive power of the surroundings $E_{b,surr}$. So, if $t_s > t_{surr}$, net heat loss from gray surface A_s in the radiation exchange with the surroundings at T_{surr} is

$$\begin{aligned} q_{net} &= q_{emitted} - q_{absorbed} = \varepsilon A_s E_{bs} - \alpha A_s E_{b,surr} \\ &= \varepsilon A_s \sigma (t_s^4 - t_{sum}^4) \end{aligned} \qquad (3)$$

where $\alpha = \varepsilon$ for the gray surface. If $t_s < t_{surr}$, the expression for q_{net} is the same with the sign reversed, and q_{net} is the net gain by A_s.

Note that q_{net} can be written as

$$q_{net} = \frac{E_{bs} - E_{b,\,surr}}{1/(\varepsilon A_s)}$$

In this form, $E_{bs} - E_{b,surr}$ is analogous to the driving potential in an electric circuit, and $1/(\varepsilon A_s)$ is analogous to electrical resistance. This is a convenient analogy when only radiation is being considered, but if convection and radiation both occur at a surface, convection is described by a driving potential based on the difference in the first power of the temperatures, whereas radiation is described by the difference in the fourth power of the temperatures. In cases like this, it is often useful to express net radiation as

$$q_{net} = h_r A_s (t_s - t_{surr}) = (t_s - t_{surr})/(1/h_r A_s) \qquad (4)$$

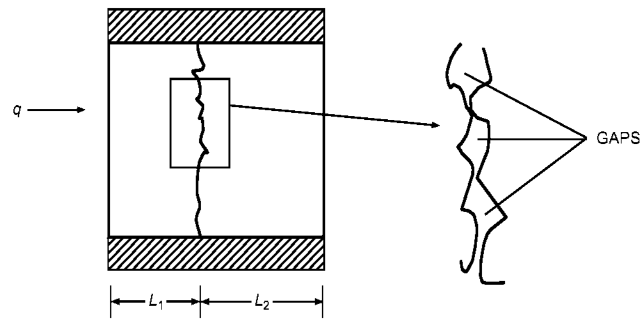


Fig. 2 Interface Resistance Across Two Layers

where $h_r = \sigma\varepsilon(t_s^2 + t_{surr}^2)(t_s + t_{surr})$ is often called a **radiation heat transfer coefficient**. The disadvantage of this form is that h_r depends on t_s, which is often the desired result of the calculation.

Combined Radiation and Convection

When $t_{surr} = t_\infty$ in Equation (4), the total heat transfer from a surface by convection and radiation combined is then

$$q = q_{rad} + q_{conv} = (t_s - t_\infty) A_s (h_r + h_c)$$

The temperature difference $t_s - t_\infty$ is in either kelvins or °C; the difference is the same. Either can be used; however, absolute temperatures *must* be used to calculate h_r. (Absolute temperatures are K = °C + 273.15.) Note that h_c and h_r are always positive, and that the direction of q is determined by the sign of $(t_s - t_\infty)$.

Contact or Interface Resistance

Heat flow through two layers encounters two conduction resistances L_1/k_1A and L_2/k_2A (Figure 2). At the interface between two layers are gaps across which heat is transferred by a combination of conduction at contact points and convection and radiation across gaps. This multimode heat transfer process is usually characterized using a contact resistance coefficient R''_{cont} or contact conductance h_{cont}.

$$q = \frac{\Delta T}{R''_{cont}/A} = h_{cont} A \Delta t$$

where Δt is the temperature drop across the interface. R''_{cont} is in (m$^2 \cdot$K)/W, and h_{cont} is in W/(m$^2 \cdot$K). The contact or interface resistance is $R_{cont} = R''_{cont}/A = 1/h_{cont}A$, and the resistance of the two layers combined is the sum of the resistances of the two layers and the contact resistance.

Contact resistance can be reduced by lowering surface roughnesses, increasing contact pressure, or using a conductive grease or paste to fill the gaps.

Heat Flux

The conduction heat transfer can be written as

$$q'' = \frac{q}{A_c} = \frac{k(t_{s1} - t_{s2})}{L}$$

where q'' is heat flux in W/m^2. Similarly, for convection the heat flux is

$$q'' = \frac{q}{A_s} = h_c(t_s - t_\infty)$$

and net heat flux from radiation at the surface is

$$q''_{net} = \frac{q_{net}}{A_s} = \varepsilon\sigma(t_s^4 - t_{surr}^4)$$

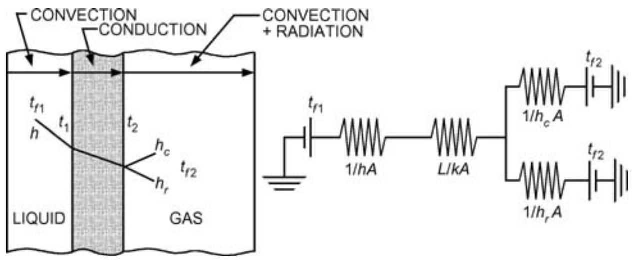


Fig. 3 Thermal Circuit

Overall Resistance and Heat Transfer Coefficient

In Equation (1) for conduction in a slab, Equation (4) for radiative heat transfer rate between two surfaces, and Equation (2) for convective heat transfer rate from a surface, the heat transfer rate is expressed as a temperature difference divided by a thermal resistance. Using the electrical resistance analogy, with temperature difference and heat transfer rate instead of potential difference and current, respectively, tools for solving series electrical resistance circuits can also be applied to heat transfer circuits. For example, consider the heat transfer rate from a liquid to the surrounding gas separated by a constant cross-sectional area solid, as shown in Figure 3. The heat transfer rate from the fluid to the adjacent surface is by convection, then across the solid body by conduction, and finally from the solid surface to the surroundings by both convection and radiation. A circuit using the equations for resistances in each mode is also shown. From the circuit, the heat transfer rate is

$$q = \frac{(t_{f1} - t_{f2})}{R_1 + R_2 + R_3}$$

where

$$R_1 = 1/hA \qquad R_2 = L/kA \qquad R_3 = \frac{(1/h_cA)(1/h_rA)}{(1/h_cA) + (1/h_rA)}$$

Resistance R_3 is the parallel combination of the convection and radiation resistances on the right-hand surface, $1/h_cA$ and $1/h_rA$. Equivalently, $R_3 = 1/h_{rc}A$, where h_{rc} on the air side is the sum of the convection and radiation heat transfer coefficients (i.e., $h_{rc} = h_c + h_r$).

The heat transfer rate can also be written as

$$q = UA(t_{f1} - t_{f2})$$

where U is the overall heat transfer coefficient that accounts for all the resistances involved. Note that

$$\frac{t_{f1} - t_{f2}}{q} = \frac{1}{UA} = R_1 + R_2 + R_3$$

The product UA is overall conductance, the reciprocal of overall resistance. The surface area A on which U is based is not always constant as in this example, and should always be specified when referring to U.

Heat transfer rates are equal from the warm liquid to the solid surface, through the solid, and then to the cool gas. Temperature drops across each part of the heat flow path are related to the resistances (as voltage drops are in an electric circuit), so that

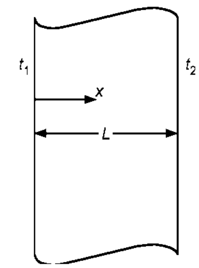
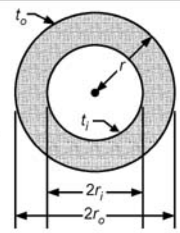
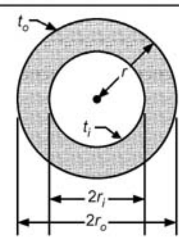
$$t_{f1} - t_1 = qR_1 \qquad t_1 - t_2 = qR_2 \qquad t_2 - t_{f2} = qR_3$$

THERMAL CONDUCTION

One-Dimensional Steady-State Conduction

Steady-state heat transfer rates and resistances for (1) a slab of constant cross-sectional area, (2) a hollow cylinder with radial heat transfer, and (3) a hollow sphere are given in Table 2.

Table 2 One-Dimensional Conduction Shape Factors

Configuration		Heat Transfer Rate	Thermal Resistance
Constant cross-sectional area slab		$q_x = kA_x \frac{t_1 - t_2}{L}$	$\frac{L}{kA_x}$
Hollow cylinder of length L with negligible heat transfer from end surfaces		$q_r = \frac{2\pi kL(t_i - t_o)}{\ln\left(\frac{r_o}{r_i}\right)}$	$R = \frac{\ln(r_o/r_i)}{2\pi kL}$
Hollow sphere		$q_r = \frac{4\pi k(t_i - t_o)}{\frac{1}{r_i} + \frac{1}{r_o}}$	$R = \frac{1/r_i - 1/r_o}{4\pi k}$

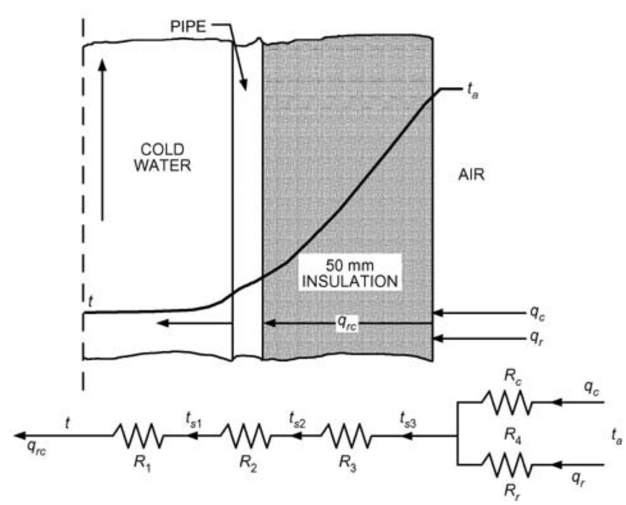


Fig. 4 Thermal Circuit Diagram for Insulated Water Pipe (Example 1)

Example 1. Chilled water at 5°C flows in a copper pipe with a thermal conductivity k_p of 400 W/(m·K), with internal and external diameters of ID = 100 mm and OD = 120 mm. The tube is covered with insulation 50 mm thick, with k_i = 0.20 W/(m·K). The surrounding air is at t_a = 25°C, and the heat transfer coefficient at the outer surface h_o = 10 W/(m^2·K). Emissivity of the outer surface is ε = 0.85. The heat transfer coefficient inside the tube is h_i = 1000 W/(m^2·K). Contact resistance between the insulation and the pipe is assumed to be negligible. Find the rate of heat gain per unit length of pipe and the temperature at the pipe-insulation interface.

Solution: The outer diameter of the insulation is D_{ins} = 120 + 2(50) = 220 mm. For L = 1 m,

$$R_1 = \frac{1}{h_i \pi \mathrm{ID} L} = 3.2 \times 10^{-3}\ \mathrm{K/W}$$

$$R_2 = \frac{\ln(\mathrm{OD/ID})}{2\pi k_p L} = 7 \times 10^{-5}\ \mathrm{K/W}$$

$$R_3 = \frac{\ln(D_{ins}/\mathrm{OD})}{2\pi k_i L} = 0.482\ \mathrm{K/W}$$

$$R_c = \frac{1}{h_o \pi D_{ins} L} = 0.144\ \mathrm{K/W}$$

Assuming insulation surface temperature $t_s = 21°C$ (i.e., 294 K) and $t_{surr} = t_a = 298.15$ K, $h_r = \varepsilon\sigma(t_s^2 + t_{surr}^2)(t_s + t_{surr}) = 5.0\ \mathrm{W/(m^2 \cdot K)}$.

$$R_r = \frac{1}{h_r \pi D_{ins} L} = 0.288\ \mathrm{K/W}$$

$$R_4 = \frac{R_r R_c}{R_r + R_c} = 0.096\ \mathrm{K/W}$$

$$R_{tot} = R_1 + R_2 + R_3 + R_4 = 0.581\ \mathrm{K/W}$$

Finally, the rate of heat gain by the cold water is

$$q_{rc} = \frac{t_a - t}{R_{tot}} = 34.4\ \mathrm{W}$$

Temperature at the pipe/insulation interface is

$$t_{s2} = t + q_{rc}(R_1 + R_2) = 5.1°C$$

Temperature at the insulation's surface is

$$t_{s3} = t_a - q_{rc}R_4 = 21.7°C$$

which is very close to the assumed value of 22°C.

Two- and Three-Dimensional Steady-State Conduction: Shape Factors

Mathematical solutions to a number of two and three-dimensional conduction problems are available in Carslaw and Jaeger (1959). Complex problems can also often be solved by graphical or numerical methods, as described by Adams and Rogers (1973), Croft and Lilley (1977), and Patankar (1980). There are many two- and three-dimensional steady-state cases that can be solved using conduction shape factors. Using the conduction shape factor S, the heat transfer rate is expressed as

$$q = Sk(t_1 - t_2) = (t_1 - t_2)/(1/Sk) \tag{5}$$

where k is the material's thermal conductivity, t_1 and t_2 are temperatures of two surfaces, and $1/(Sk)$ is thermal resistance. Conduction shape factors for some common configurations are given in Table 3.

Example 2. The walls and roof of a house are made of 200 mm thick concrete with $k = 0.75\ \mathrm{W/(m \cdot K)}$. The inner surface is at 20°C, and the outer surface is at 8°C. The roof is 10 × 10 m, and the walls are 6 m high. Find the rate of heat loss from the house through its walls and roof, including edge and corner effects.

Solution: The rate of heat transfer excluding the edges and corners is first determined:

$$A_{total} = (10 - 0.4)(10 - 0.4) + 4(10 - 0.4)(6 - 0.2) = 314.9\ \mathrm{m^2}$$

$$q_{walls-ceiling} = \frac{kA_{total}}{L}\Delta T = \frac{[0.75\ \mathrm{W/(m \cdot K)}](314.9\ \mathrm{m^2})}{0.2\ \mathrm{m}}(20 - 8)°C = 14\,170\ \mathrm{W}$$

The shape factors for the corners and edges are in Table 2:

$$S_{corners+edges} = 4 \times S_{corner} + 4 \times S_{edge} = 4 \times 0.15L + 4 \times 0.54\mathrm{W} = 4 \times 0.15(0.2\ \mathrm{m}) + 4 \times 0.54(9.6\ \mathrm{m}) = 20.86\ \mathrm{m}$$

and the heat transfer rate is

$$q_{corners-edges} = S_{corners+edges}k\Delta T = (20.86\ \mathrm{m})[0.75\ \mathrm{W/(m \cdot K)}](20 - 8)°C = 188\ \mathrm{W}$$

which leads to

$$q_{total} = 14\,170\ \mathrm{W} + 188\ \mathrm{W} = 14\,358\ \mathrm{W} = 14.4\ \mathrm{kW}$$

Note that the edges and corners are 1.3% of the total.

Extended Surfaces

Heat transfer from a surface can be increased by attaching fins or extended surfaces to increase the area available for heat transfer. A few common fin geometries are shown in Figures 5 to 8. Fins provide a large surface area in a low volume, thus lowering material costs for a given performance. To achieve optimum design, fins are generally located on the side of the heat exchanger with lower heat transfer coefficients (e.g., the air side of an air-to-water coil). Equipment with extended surfaces includes natural- and forced-convection coils and shell-and-tube evaporators and condensers. Fins are also used inside tubes in condensers and dry expansion evaporators.

Fin Efficiency. As heat flows from the root of a fin to its tip, temperature drops because of the fin material's thermal resistance. The temperature difference between the fin and surrounding fluid is therefore greater at the root than at the tip, causing a corresponding variation in heat flux. Therefore, increases in fin length result in proportionately less additional heat transfer. To account for this effect, **fin efficiency** ϕ is defined as the ratio of the actual heat transferred from the fin to the heat that would be transferred if the entire fin were at its root or base temperature:

$$\phi = \frac{q}{hA_s(t_r - t_e)} \tag{6}$$

where q is heat transfer rate into/out of the fin's root, t_e is temperature of the surrounding environment, t_r is temperature at fin root, and A_s is surface area of the fin. Fin efficiency is low for long or thin fins, or fins made of low-thermal-conductivity material. Fin efficiency decreases as the heat transfer coefficient increases because of increased heat flow. For natural convection in air-cooled condensers and evaporators, where the air-side h is low, fins can be fairly large and fabricated from low-conductivity materials such as steel instead of from copper or aluminum. For condensing and boiling, where large heat transfer coefficients are involved, fins must be very short for optimum use of material. Fin efficiencies for a few geometries are shown in Figures 5 to 8. Temperature distribution and fin efficiencies for various fin shapes are derived in most heat transfer texts.

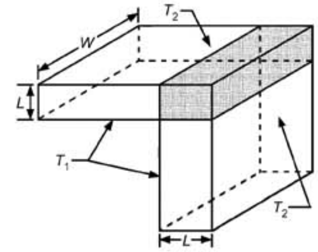
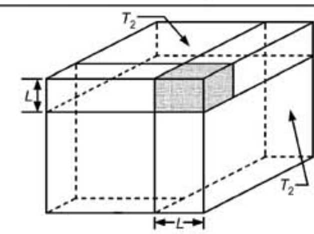
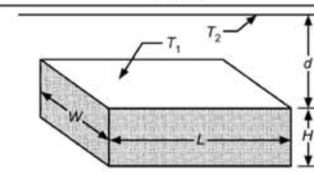
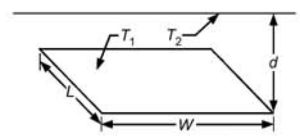
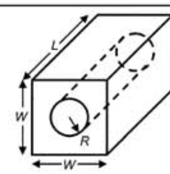
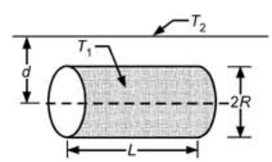
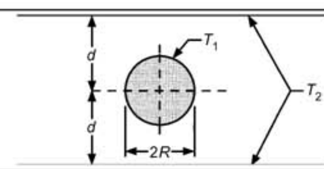
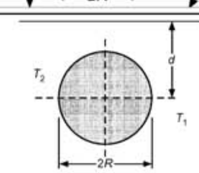
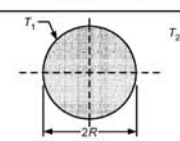
Constant-Area Fins and Spines. Fins or spines with constant cross-sectional area [e.g., straight fins (option A in Figure 7), cylindrical spines (option D in Figure 8)], the efficiency can be calculated as

$$\phi = \frac{\tanh(mW_c)}{mW_c} \tag{7}$$

where

$m = \sqrt{hP/kA_c}$
P = fin perimeter
A_c = fin cross-sectional area
W_c = corrected fin/spine length = $W + A_c/P$
A_c/P = $d/4$ for a cylindrical spine with diameter d
= $a/4$ for an $a \times a$ square spine
= $y_b = \delta/2$ for a straight fin with thickness δ

Table 3 Multidimensional Conduction Shape Factors

Configuration	Shape Factor S, m	Restriction	
Edge of two adjoining walls	$0.54W$	$W > L/5$	
Corner of three adjoining walls (inner surface at T_1 and outer surface at T_2)	$0.15L$	$L \ll$ length and width of wall	
Isothermal rectangular block embedded in semi-infinite body with one face of block parallel to surface of body	$\dfrac{2.756L}{\left[\ln\left(1+\dfrac{d}{W}\right)\right]^{0.59}}\left(\dfrac{H}{d}\right)^{0.078}$	$L > W$ $L >> d, W, H$	
Thin isothermal rectangular plate buried in semi-infinite medium	$\dfrac{\pi W}{\ln(4W/L)}$ $\dfrac{2\pi W}{\ln(4W/L)}$ $\dfrac{2\pi W}{\ln(2\pi d/L)}$	$d = 0,\ W > L$ $d >> W$ $W > L$ $d > 2W$ $W >> L$	
Cylinder centered inside square of length L	$\dfrac{2\pi L}{\ln(0.54W/R)}$	$L >> W$ $W > 2R$	
Isothermal cylinder buried in semi-infinite medium	$\dfrac{2\pi L}{\cosh^{-1}(d/R)}$ $\dfrac{2\pi L}{\ln(2d/R)}$ $\dfrac{2\pi L}{\ln\dfrac{L}{R}\left[1-\dfrac{\ln(L/2d)}{\ln(L/R)}\right]}$	$L >> R$ $L >> R$ $d > 3R$ $d >> R$ $L >> d$	
Horizontal cylinder of length L midway between two infinite, parallel, isothermal surfaces	$\dfrac{2\pi L}{\ln\left(\dfrac{4d}{R}\right)}$	$L >> d$	
Isothermal sphere in semi-infinite medium	$\dfrac{4\pi R}{1-(R/2d)}$		
Isothermal sphere in infinite medium	$4\pi R$		

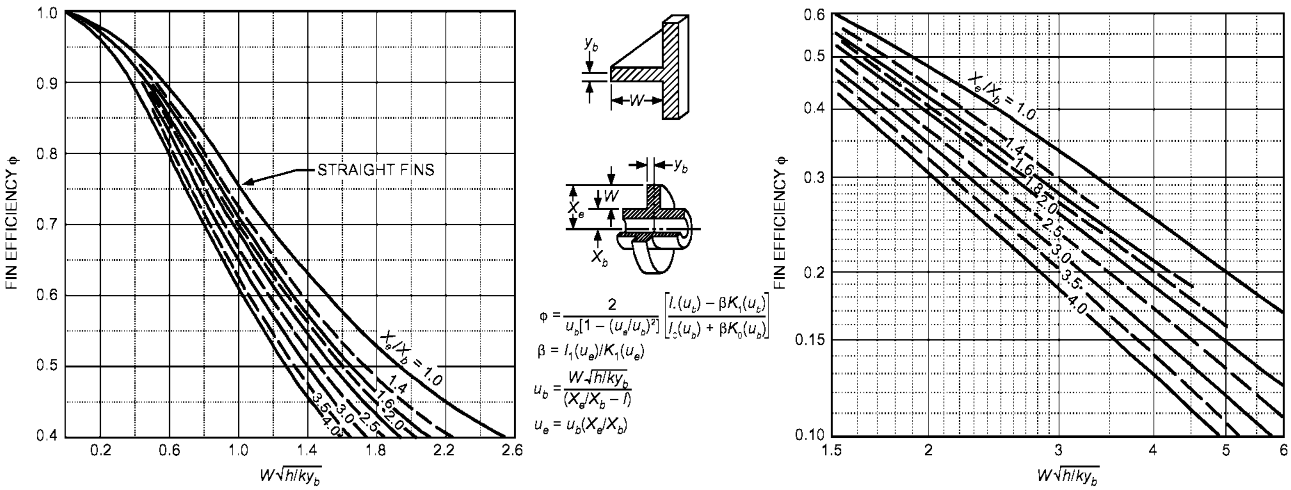


Fig. 5 Efficiency of Annular Fins of Constant Thickness

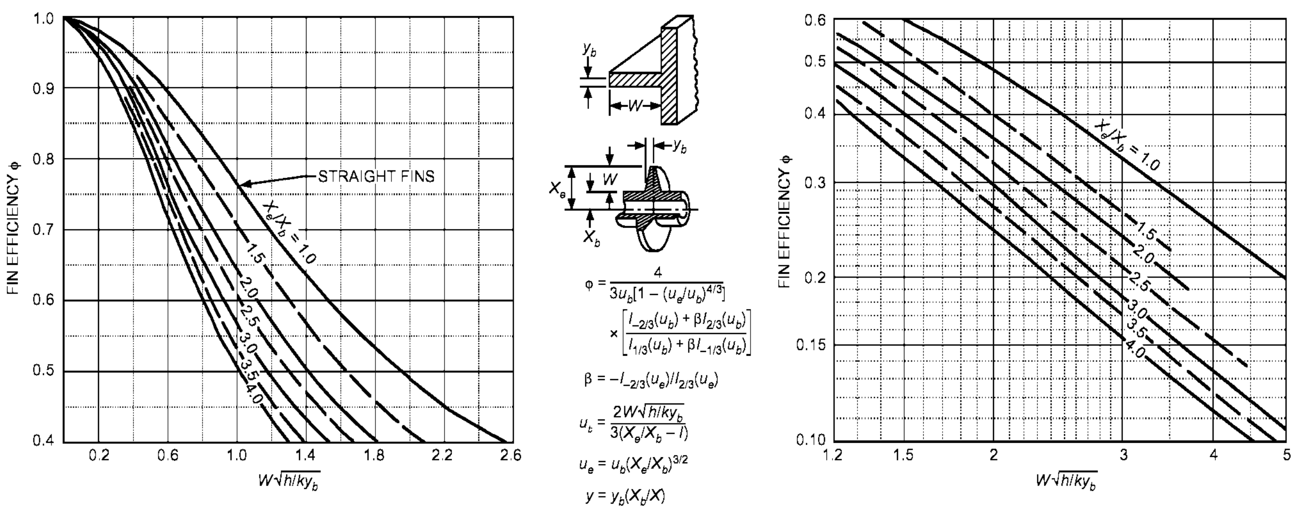


Fig. 6 Efficiency of Annular Fins with Constant Metal Area for Heat Flow

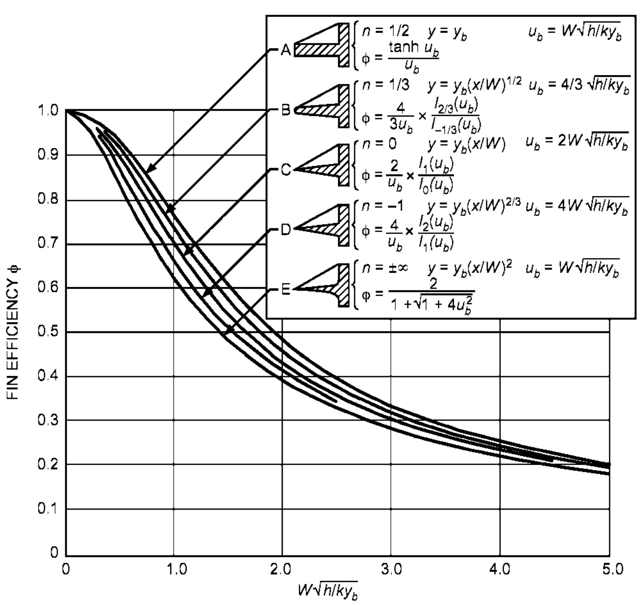


Fig. 7 Efficiency of Several Types of Straight Fins

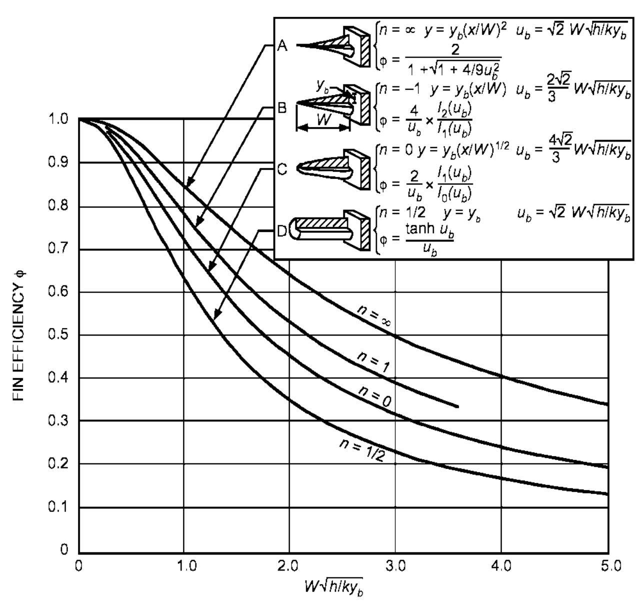


Fig. 8 Efficiency of Four Types of Spines

Empirical Expressions for Fins on Tubes. Schmidt (1949) presents approximate, but reasonably accurate, analytical expressions (for computer use) for the fin efficiency of circular, rectangular, and hexagonal arrays of fins on round tubes, as shown in Figures 5, 9, and 10, respectively. Rectangular fin arrays are used for an in-line tube arrangement in finned-tube heat exchangers, and hexagonal arrays are used for staggered tubes. Schmidt's empirical solution is given by

$$\phi = \frac{\tanh(mr_bZ)}{mr_bZ} \tag{8}$$

where r_b is tube radius, $m = \sqrt{2h/k\delta}$, δ = fin thickness, and Z is given by

$$Z = [(r_e/r_b) - 1][1 + 0.35\ln(r_e/r_b)]$$

where r_e is the actual or equivalent fin tip radius. For **circular fins**, r_e/r_b is the actual ratio of fin tip radius to tube radius. For rectangular fins (Figure 9),

$$r_e/r_b = 1.28\Psi\sqrt{\beta - 0.2} \qquad \Psi = M/r_b \qquad \beta = L/M \ge 1$$

where M and L are defined by Figure 9 as $a/2$ or $b/2$, depending on which is greater. For hexagonal fins (Figure 10),

$$r_e/r_b = 1.27\Psi\sqrt{\beta - 0.3}$$

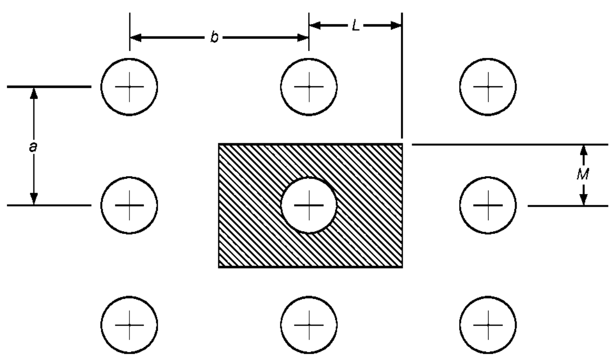


Fig. 9 Rectangular Tube Array

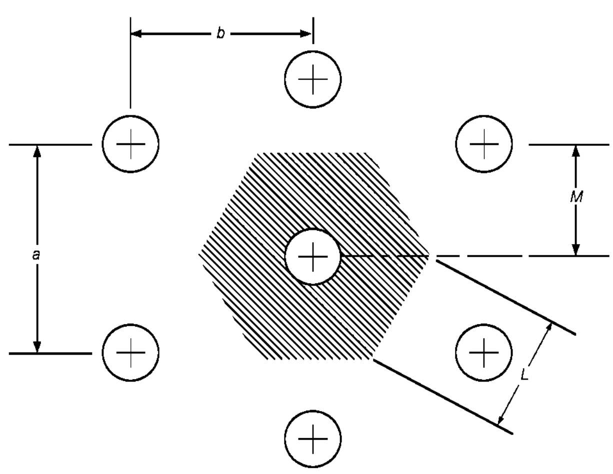


Fig. 10 Hexagonal Tube Array

where Ψ and β are defined as previously, and M and L are defined by Figure 10 as $a/2$ or b (whichever is less) and $0.5\sqrt{(a/2)^2 + b^2}$, respectively.

For constant-thickness square fins on a round tube ($L = M$ in Figure 9), the efficiency of a constant-thickness annular fin of the same area can be used. For more accuracy, particularly with rectangular fins of large aspect ratio, divide the fin into circular sectors as described by Rich (1966).

Other sources of information on finned surfaces are listed in the References and Bibliography.

Surface Efficiency. Heat transfer from a finned surface (e.g., a tube) that includes both fin area A_s and unfinned or prime area A_p is given by

$$q = (h_pA_p + \phi h_sA_s)(t_r - t_e) \tag{9}$$

Assuming the heat transfer coefficients for the fin and prime surfaces are equal, a **surface efficiency** ϕ_s can be derived as

$$\phi_s = \frac{A_p + \phi A_s}{A} \tag{10}$$

where $A = A_s + A_p$ is the total surface area, the sum of the fin and prime areas. The heat transfer in Equation (8) can then be written as

$$q = \phi_s hA(t_r - t_e) = \frac{t_r - t_e}{1/(\phi_s hA)} \tag{11}$$

where $1/(\phi_s hA)$ is the finned surface resistance.

Example 3. An aluminum tube with k = 186 W/(m·K), ID = 45 mm, and OD = 50 mm has circular aluminum fins δ = 1 mm thick with an outer diameter of D_{fin} = 100 mm. There are N' = 250 fins per metre of tube length. Steam condenses inside the tube at t_i = 200°C with a large heat transfer coefficient on the inner tube surface. Air at t_∞ = 25°C is heated by the steam. The heat transfer coefficient outside the tube is 40 W/(m²·K). Find the rate of heat transfer per metre of tube length.

Solution: From Figure 5's efficiency curve, the efficiency of these circular fins is

$$\left.\begin{array}{l} W = (D_{fin} - \text{OD})/2 = (0.10 - 0.05)/2 = 0.025\text{ m} \\ X_e/X_b = 0.10/0.05 = 2.0 \\ W\sqrt{\dfrac{h}{k(\delta/2)}} = 0.025\sqrt{\dfrac{40\text{ W/(m}^2\cdot\text{K)}}{[186\text{ W/(m}\cdot\text{K)}](0.0005\text{ m})}} = 0.52 \end{array}\right\} \phi = 0.89$$

The fin area for L = 1 m is

$$A_s = 250 \times 2\pi(D_{fin}^2 - \text{OD}^2)/4 = 2.945\text{ m}^2$$

The unfinned area for L = 1 m is

$$A_p = \pi \times \text{OD} \times L(1 - N'\delta) = \pi(0.05\text{ m})(1\text{ m})(1 - 250 \times 0.001) = 0.118\text{ m}^2$$

and the total area $A = A_s + A_p = 3.063\text{ m}^2$. Surface efficiency is

$$\phi_s = \frac{\phi A_f + A_s}{A} = 0.894$$

and resistance of the finned surface is

$$R_s = \frac{1}{\phi_s hA} = 9.13 \times 10^{-3}\text{ K/W}$$

Tube wall resistance is

$$R_{wall} = \frac{\ln(\text{OD/ID})}{2\pi Lk_{tube}} = \frac{\ln(5/4.5)}{2\pi(1\text{ m})[186\text{ W/(m}\cdot\text{K)}]} = 9.02 \times 10^{-5}\text{ K/W}$$

The rate of heat transfer is then

$$q = \frac{t_i - t_\infty}{R_s + R_{wall}} = 18\ 981\ \text{W}$$

Had Schmidt's approach been used for fin efficiency,

$$m = \sqrt{2h/k\delta} = 20.74\ \text{m}^{-1} \qquad r_b = \text{OD}/2 = 0.025\ \text{m}$$

$$Z = [(D_{fin}/\text{OD}) - 1]\ [1 + 0.35\ \ln(D_{fin}/\text{OD})] = 1.243$$

$$\phi = \frac{\tanh(mr_bZ)}{mr_bZ} = 0.88$$

the same ϕ as given by Figure 5.

Contact Resistance. Fins can be extruded from the prime surface (e.g., short fins on tubes in flooded evaporators or water-cooled condensers) or can be fabricated separately, sometimes of a different material, and bonded to the prime surface. Metallurgical bonds are achieved by furnace-brazing, dip-brazing, or soldering; nonmetallic bonding materials, such as epoxy resin, are also used. Mechanical bonds are obtained by tension-winding fins around tubes (spiral fins) or expanding the tubes into the fins (plate fins). Metallurgical bonding, properly done, leaves negligible thermal resistance at the joint but is not always economical. Contact resistance of a mechanical bond may or may not be negligible, depending on the application, quality of manufacture, materials, and temperatures involved. Tests of plate-fin coils with expanded tubes indicate that substantial losses in performance can occur with fins that have cracked collars, but negligible contact resistance was found in coils with continuous collars and properly expanded tubes (Dart 1959).

Contact resistance at an interface between two solids is largely a function of the surface properties and characteristics of the solids, contact pressure, and fluid in the interface, if any. Eckels (1977) modeled the influence of fin density, fin thickness, and tube diameter on contact pressure and compared it to data for wet and dry coils. Shlykov (1964) showed that the range of attainable contact resistances is large. Sonokama (1964) presented data on the effects of contact pressure, surface roughness, hardness, void material, and the pressure of the gas in the voids. Lewis and Sauer (1965) showed the resistance of adhesive bonds, and Clausing (1964) and Kaspareck (1964) gave data on the contact resistance in a vacuum environment.

Transient Conduction

Often, heat transfer and temperature distribution under transient (i.e., varying with time) conditions must be known. Examples are (1) cold-storage temperature variations on starting or stopping a refrigeration unit, (2) variation of external air temperature and solar irradiation affecting the heat load of a cold-storage room or wall temperatures, (3) the time required to freeze a given material under certain conditions in a storage room, (4) quick-freezing objects by direct immersion in brines, and (5) sudden heating or cooling of fluids and solids from one temperature to another.

Lumped Mass Analysis. Often, the temperature within a mass of material can be assumed to vary with time but be uniform within the mass. Examples include a well-stirred fluid in a thin-walled container, or a thin metal plate with high thermal conductivity. In both cases, if the mass is heated or cooled at its surface, the temperature can be assumed to be a function of time only and not location within the body. Such an approximation is valid if

$$\text{Bi} = \frac{h(V/A_s)}{k} \le 0.1$$

where

Bi = Biot number
h = surface heat transfer coefficient
V = material's volume
A_s = surface area exposed to convective and/or radiative heat transfer
k = material's thermal conductivity

The temperature is given by

$$Mc_p\frac{dt}{d\tau} = q_{net} + q_{gen} \qquad (12)$$

where

M = body mass
c_p = specific heat
q_{gen} = internal heat generation
q_{net} = net heat transfer rate to substance (into substance is positive, and out of substance is negative)

Equation (12) applies to liquids and solids. If the material is a gas being heated or cooled at constant volume, replace c_p with the constant-volume specific heat c_v. The term q_{net} may include heat transfer by conduction, convection, or radiation and is the difference between the heat transfer rates into and out of the body. The term q_{gen} may include a chemical reaction (e.g., curing concrete) or heat generation from a current passing through a metal.

For a lumped mass M initially at a uniform temperature t_0 that is suddenly exposed to an environment at a different temperature t_∞, the time taken for the temperature of the mass to change to t_f is given by the solution of Equation (12) as

$$\ln\frac{t_f - t_\infty}{t_0 - t_\infty} = -\frac{hA_s\tau}{Mc_p} \qquad (13)$$

where

M = mass of solid
c_p = specific heat of solid
A_s = surface area of solid
h = surface heat transfer coefficient
τ = time required for temperature change
t_f = final solid temperature
t_0 = initial uniform solid temperature
t_∞ = surrounding fluid temperature

Example 4. A copper sphere with diameter d = 1 mm is to be used as a sensing element for a thermostat. It is initially at a uniform temperature of t_0 = 21°C. It is then exposed to the surrounding air at t_∞ = 20°C. The combined heat transfer coefficient is h = 60 W/(m²·K). Determine the time taken for the temperature of the sensing element to reach t_f = 20.5°C. The properties of copper are

$$\rho = 8933\ \text{kg/m}^3 \qquad c_p = 385\ \text{J/(kg·K)} \qquad k = 401\ \text{W/(m·K)}$$

Solution: Bi = $h(d/2)/k$ = 60.35(0.001/2)/401 = 7.5 × 10^{-5}, which is much less than 1. Therefore, lumped analysis is valid.

$$M = \rho[4\pi(d/2)^3/3] = 4.677 \times 10^{-6}\ \text{kg}$$

$$A_s = \pi d^2 = 3.142 \times 10^{-6}\ \text{m}^2$$

Using Equation (13), τ = 6.6 s.

Nonlumped Analysis. When the Biot number is greater than 0.1, variation of temperature with location within the mass is significant. One example is the cooling time of meats in a refrigerated space: the meat's size and conductivity do not allow it to be treated as a lumped mass that cools uniformly. Nonlumped problems require solving multidimensional partial differential equations. Many common cases have been solved and presented in graphical forms (Jakob 1949, 1957; Myers 1971; Schneider 1964). In other cases, numerical methods (Croft and Lilley 1977; Patankar 1980) must be used.

Estimating Cooling Times for One-Dimensional Geometries. When a slab of thickness $2L$ or a solid cylinder or solid sphere with outer radius r_m is initially at a uniform temperature t_1, and its surface is suddenly heated or cooled by convection with a fluid at t_∞, a mathematical solution is available for the temperature t as a function of

Table 4 Values of c_1 and μ_1 in Equations (14) to (17)

	Slab		Solid Cylinder		Solid Sphere	
Bi	c_1	μ_1	c_1	μ_1	c_1	μ_1
0.5	1.0701	0.6533	1.1143	0.9408	1.1441	1.1656
1.0	1.1191	0.8603	1.2071	1.2558	1.2732	1.5708
2.0	1.1785	1.0769	1.3384	1.5995	1.4793	2.0288
4.0	1.2287	1.2646	1.4698	1.9081	1.7202	2.4556
6.0	1.2479	1.3496	1.5253	2.0490	1.8338	2.6537
8.0	1.2570	1.3978	1.5526	2.1286	1.8920	2.7654
10.0	1.2620	1.4289	1.5677	2.1795	1.9249	2.8363
30.0	1.2717	1.5202	1.5973	2.3261	1.9898	3.0372
50.0	1.2727	1.5400	1.6002	2.3572	1.9962	3.0788

location and time τ. The solution is an infinite series. However, after a short time, the temperature is very well approximated by the first term of the series. The single-term approximations for the three cases are of the form

$$Y = Y_0 f(\mu_1 n) \tag{14}$$

where

$$Y = \frac{t - t_\infty}{t_1 - t_\infty}$$

$$Y_0 = \frac{t_0 - t_\infty}{t_1 - t_\infty} = c_1 \exp(-\mu_1^2 \mathrm{Fo})$$

t_0 = temperature at center of slab, cylinder, or sphere
Fo = $\alpha\tau/L_c^2$ = Fourier number
α = thermal diffusivity of solid = $k/\rho c_p$
L_c = L for slab, r_o for cylinder, sphere
n = x/L for slab, r/r_m for cylinder
c_1, μ_1 = coefficients that are functions of Bi
Bi = Biot number = hL_c/k
$f(\mu_1 n)$ = function of $\mu_1 n$, different for each geometry
x = distance from midplane of slab of thickness $2L$ cooled on both sides
ρ = density of solid
c_p = constant pressure specific heat of solid
k = thermal conductivity of solid

The single term solution is valid for Fo > 0.2. Values of c_1 and μ_1 are given in Table 4 for a few values of Bi, and Couvillion (2004) provides a procedure for calculating them. Expressions for c_1 for each case, along with the function $f(\mu_1 n)$, are as follows:

Slab

$$f(\mu_1 n) = \cos(\mu_1 n) \qquad c_1 = \frac{4\sin(\mu_1)}{2\mu_1 + \sin(2\mu_1)} \tag{15}$$

Long solid cylinder

$$f(\mu_1 n) = J_0(\mu_1 n) \qquad c_1 = \frac{2}{\mu_1} \times \frac{J_1(\mu_1)}{J_0^2(\mu_1) + J_1^2(\mu_1)} \tag{16}$$

where J_0 is the Bessel function of the first kind, order zero. It is available in math tables, spreadsheets, and software packages. $J_0(0) = 1$.

Solid sphere

$$f(\mu_1 n) = \frac{\sin(\mu_1 n)}{\mu_1 n} \qquad c_1 = \frac{4[\sin(\mu_1) - \mu_1\cos(\mu_1)]}{2\mu_1 - \sin(2\mu_1)} \tag{17}$$

These solutions are presented graphically (McAdams 1954) by Gurnie-Lurie charts (Figures 11 to 13). The charts are also valid for Fo < 0.2.

Example 5. Apples, approximated as 0.60 mm diameter solid spheres and initially at 30°C, are loaded into a chamber maintained at 0°C. If the surface heat transfer coefficient h = 14 W/(m^2·K), estimate the time required for the center temperature to reach t = 1°C.

Properties of apples are

$$\rho = 830 \text{ kg/m}^3 \qquad k = 0.42 \text{ W/(m}^2\cdot\text{K)}$$
$$c_p = 3600 \text{ J/(kg}\cdot\text{K)} \qquad r_m = d/2 = 30 \text{ mm} = 0.03 \text{ m}$$

Solution: Assuming that it will take a long time for the center temperature to reach 1°C, use the one-term approximation Equation (14). From the values given,

$$Y = \frac{t_\infty - t}{t_\infty - t_1} = \frac{0-1}{0-30} = \frac{1}{30}$$

$$n = \frac{r}{r_m} = \frac{0}{0.03} = 0 \qquad \mathrm{Bi} = \frac{hr_m}{k} = \frac{14 \times 0.03}{0.42} = 1$$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.42}{830 \times 3600} = 1.406 \times 10^{-7} \text{ m}^2/\text{s}$$

From Equations (14) and (17) with lim(sin 0/0) = 1, $Y = Y_0 = c_1\exp(-\mu^2{}_1\mathrm{Fo})$. For Bi = 1, from Table 4, c_1= 1.2732 and μ_1 = 1.5708. Thus,

$$\mathrm{Fo} = -\frac{1}{\mu_1^2}\ln\frac{Y}{c_1} = -\frac{1}{1.5708^2}\ln 0.0333 = 1.476 = \frac{\alpha\tau}{r_m^2} = \frac{0.00545\tau}{(0.1967/2)^2}$$

$$\tau = 2.62 \text{ h}$$

Note that Fo = 0.2 corresponds to an actual time of 1280 s.

Multidimensional Cooling Times. One-dimensional transient temperature solutions can be used to find the temperatures with two- and three-dimensional temperatures of solids. For example, consider a solid cylinder of length $2L$ and radius r_m exposed to a fluid at t_c on all sides with constant surface heat transfer coefficients h_1 on the end surfaces and h_2 on the cylindrical surface, as shown in Figure 14.

The two-dimensional, dimensionless temperature $Y(x_1,r_1,\tau)$ can be expressed as the product of two one-dimensional temperatures $Y_1(x_1,\tau) \times Y_2(r_1,\tau)$, where

Y_1 = dimensionless temperature of constant cross-sectional area slab at (x_1,τ), with surface heat transfer coefficient h_1 associated with two parallel surfaces
Y_2 = dimensionless temperature of solid cylinder at (r_1,τ) with surface heat transfer coefficient h_2 associated with cylindrical surface

From Figures 11 and 12 or Equations (14) to (16), determine Y_1 at $(x_1/L, \alpha\tau/L^2, h_1L/k)$ and Y_2 at $(r_1/r_m, \alpha\tau/r^2{}_m, h_2r_m/k)$.

Example 6. A 70 mm diameter by 125 mm high soda can, initially at t_1 = 30°C, is cooled in a chamber where the air is at t_∞ = 0°C. The heat transfer coefficient on all surfaces is h = 20 W/(m^2·K). Determine the maximum temperature in the can τ = 1 h after starting the cooling. Assume the properties of the soda are those of water, and that the soda inside the can behaves as a solid body.

Solution: Because the cylinder is short, the temperature of the soda is affected by the heat transfer rate from the cylindrical surface and end surfaces. The slowest change in temperature, and therefore the maximum temperature, is at the center of the cylinder. Denoting the dimensionless temperature by Y,

$$Y = Y_{cyl} \times Y_{pl}$$

where Y_{cyl} is the dimensionless temperature of an infinitely long 70 mm diameter cylinder, and Y_{pl} is the dimensionless temperature of a

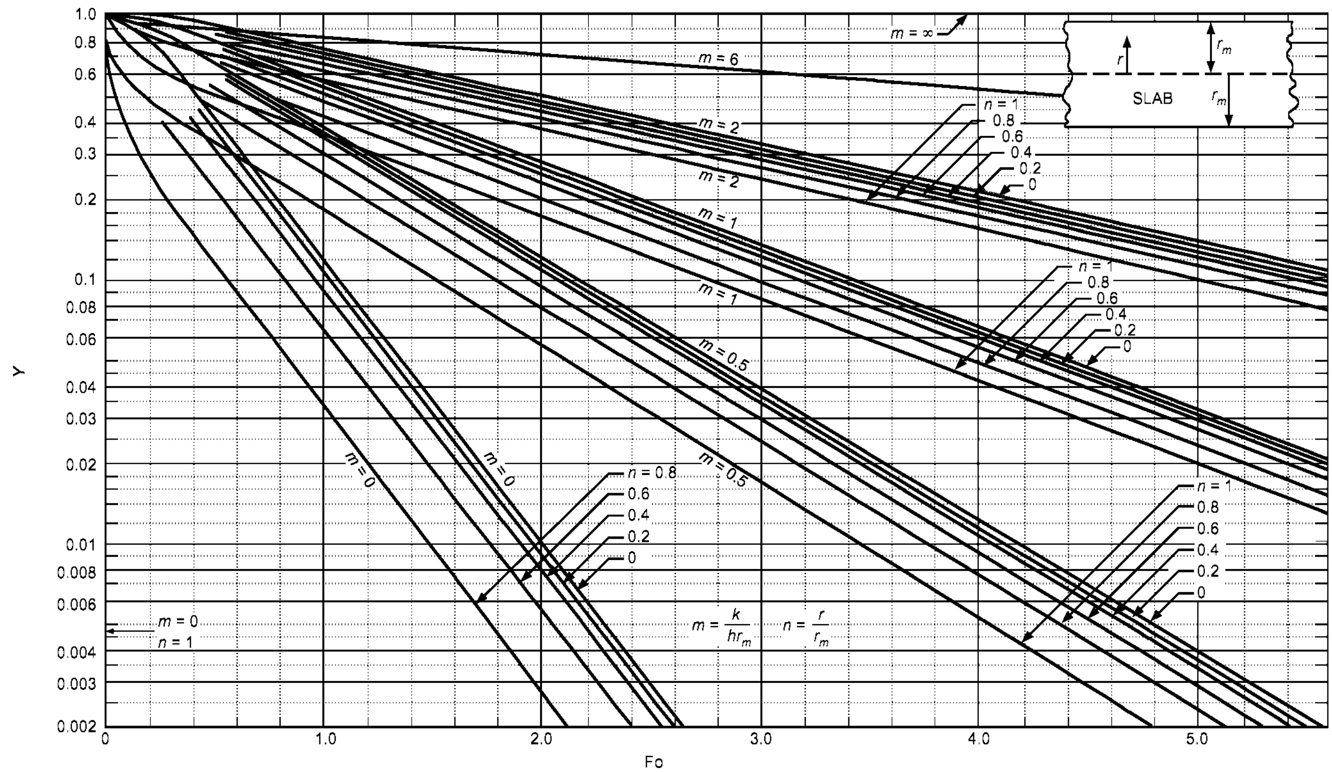


Fig. 11 Transient Temperatures for Infinite Slab, *m* = 1/Bi

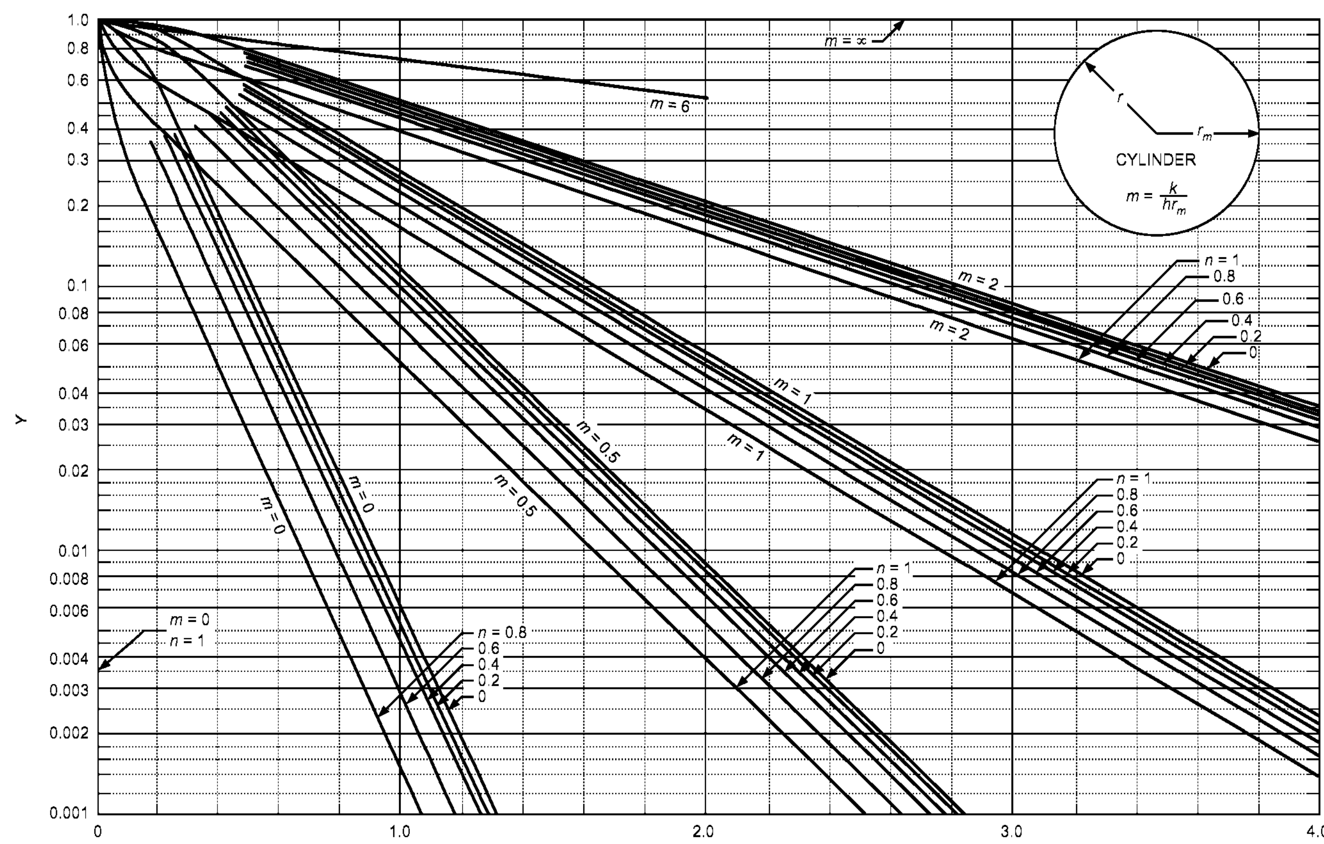


Fig. 12 Transient Temperatures for Infinite Cylinder, *m* = 1/Bi

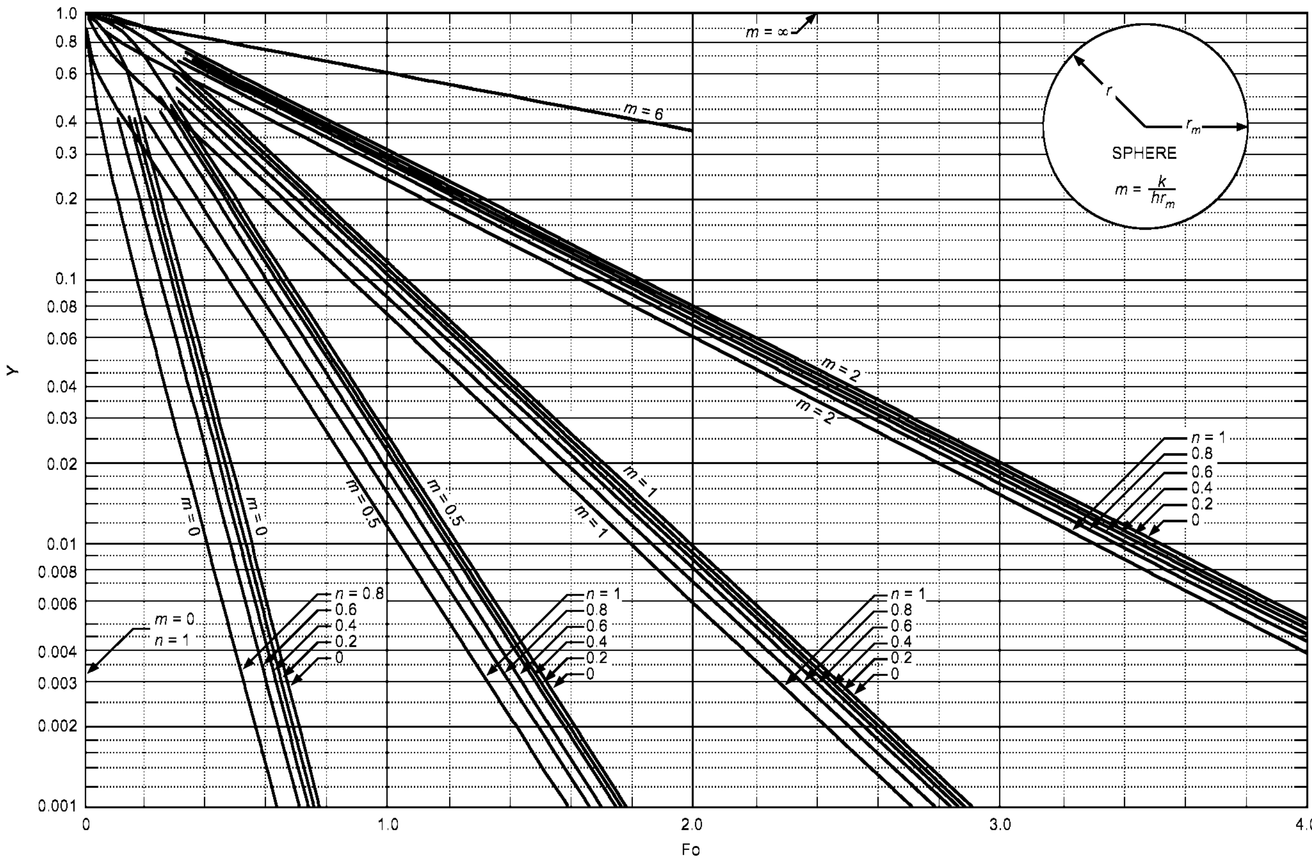


Fig. 13 Transient Temperatures for Sphere, m = 1/Bi

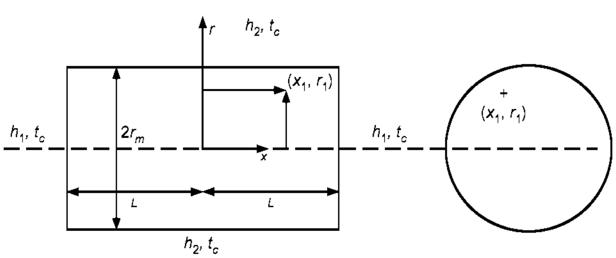


Fig. 14 Solid Cylinder Exposed to Fluid

125 mm thick slab. Each of them is found from the appropriate Biot and Fourier number. For evaluating the properties of water, choose a temperature of 15°C and a pressure of 101.35 kPa. The properties of water are

$\rho = 999.1$ kg/m^3 $\quad k = 0.5894$ W/(m·K) $\quad c_p = 4184$ J/(kg·K)

$\alpha = k/\rho = 1.41 \times 10^{-7}$ m^2/s $\quad \tau = 3600$ s

1. Determine Y_{cyl} at $n = 0$.

$$\mathrm{Bi}_{cyl} = hr_m/k = 20 \times 0.035/0.5894 = 1.188$$

$$\mathrm{Fo}_{cyl} = \alpha\tau/r_m^2 = (1.41 \times 10^{-7}) \times 3600/0.035^2 = 0.4144$$

$\mathrm{Fo}_{cyl} > 0.2$, so use the one-term approximation with Equations (14) and (16).

$$Y_{cyl} = c_1 \exp(-\mu^2{}_1\mathrm{Fo}_{cyl})J_0(0)$$

Interpolating in Table 4 for $\mathrm{Bi}_{cyl} = 1.188$, $\mu_{cyl} = 1.3042$, $J_0(0) = 1$, $c_{cyl} = 1.237$, $Y_{cyl} = 0.572$.

2. Determine Y_{pl} at $n = 0$.

$$\mathrm{Bi}_{pl} = hL/k = 20 \times 0.0625/0.5894 = 2.121$$

$$\mathrm{Fo}_{pl} = 1.41 \times 10^{-7} \times 3600/0.0625^2 = 0.1299$$

$\mathrm{Fo}_{pl} < 0.2$, so the one-term approximation is not valid. Using Figure 11, $Y_{pl} = 0.9705$. Thus,

$$Y = 0.572 \times 0.9705 = 0.5551 = (t - t_\infty)/(t_1 - t_\infty) \Rightarrow t = 16.7°\mathrm{C}$$

Note: The solution may not be exact because convective motion of the soda during heat transfer has been neglected. The example illustrates the use of the technique. For well-stirred soda, with uniform temperature within the can, the lumped mass solution should be used.

THERMAL RADIATION

Radiation, unlike conduction and convection, does not need a solid or fluid to transport energy from a high-temperature surface to a lower-temperature one. (Radiation is in fact impeded by such a material.) The rate of radiant energy emission and its characteristics from a surface depend on the underlying material's nature, microscopic arrangement, and absolute temperature. The rate of emission from a surface is independent of the surfaces surrounding it, but the rate and characteristics of radiation incident on a surface do depend on the temperatures and spatial relationships of the surrounding surfaces.

Blackbody Radiation

The total energy emitted per unit time per unit area of a black surface is called the **blackbody emissive power W_b** and is given by the **Stefan-Boltzmann law**:

$$W_b = \sigma T^4 \tag{18}$$

where $\sigma = 5.670 \times 10^{-8}$ W/(m²·K⁴) is the Stefan-Boltzmann constant.

Energy is emitted in the form of photons or electromagnetic waves of many different frequencies or wavelengths. Planck showed that the spectral distribution of the energy radiated by a blackbody is

$$W_{b\lambda} = \frac{C_1}{\lambda^5(e^{C_2/\lambda T} - 1)} \tag{19}$$

where

$W_{b\lambda}$ = blackbody spectral (monochromatic) emissive power, W/m³
λ = wavelength, m
T = temperature, K
C_1 = first Planck's law constant = 3.742×10^{-16} W·m²
C_2 = second Planck's law constant = 0.014 388 m·K

The **blackbody spectral emissive power** $W_{b\lambda}$ is the energy emitted per unit time per unit surface area at wavelength λ per unit wavelength band around λ; that is, the energy emitted per unit time per unit surface area in the wavelength band $d\lambda$ is equal to $W_{b\lambda}d\lambda$. The Stefan-Boltzmann law can be obtained by integrating Equation (19) over all wavelengths:

$$\int_0^\infty W_{b\lambda}d\lambda = \sigma T^4 = W_b$$

Wien showed that the wavelength λ_{max}, at which the monochromatic emissive power is a maximum (not the maximum wavelength), is given by

$$\lambda_{max}T = 2898\ \mu\text{m}\cdot\text{K} \tag{20}$$

Equation (20) is **Wien's displacement** law; the maximum spectral emissive power shifts to shorter wavelengths as temperature increases, such that, at very high temperatures, significant emission eventually occurs over the entire visible spectrum as shorter wavelengths become more prominent. For additional details, see Incropera et al. (2007).

Actual Radiation

The blackbody emissive power W_b and blackbody spectral emissive power $W_{b\lambda}$ are the maxima at a given surface temperature. Actual surfaces emit less and are called **nonblack**. The **emissive power** W of a nonblack surface at temperature T radiating to the hemispherical region above it is given by

$$W = \varepsilon\sigma T^4 \tag{21}$$

where ε is the **total emissivity**. The **spectral emissive power** W_λ of a nonblack surface is given by

$$W_\lambda = \varepsilon_\lambda W_{b\lambda} \tag{22}$$

where ε_λ is the **spectral emissivity**, and $W_{b\lambda}$ is given by Equation (19). The relationship between ε and ε_λ is given by

$$W = \varepsilon\sigma T^4 = \int_0^\infty W_\lambda d\lambda = \int_0^\infty \varepsilon_\lambda W_{b\lambda}d\lambda$$

or

$$\varepsilon = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda W_{b\lambda}d\lambda \tag{23}$$

If ε_λ does not depend on λ, then, from Equation (23), $\varepsilon = \varepsilon_\lambda$, and the surface is called **gray**. Gray surface characteristics are often assumed in calculations. Several classes of surfaces approximate this condition in some regions of the spectrum. The simplicity is desirable, but use care, especially if temperatures are high. Grayness is sometimes assumed because of the absence of information relating ε_λ as a function of λ.

Emissivity is a function of the material, its surface condition, and its surface temperature. Table 5 lists selected values; Modest (2003) and Siegel and Howell (2002) have more extensive lists.

When radiant energy reaches a surface, it is absorbed, reflected, or transmitted through the material. Therefore, from the first law of thermodynamics,

$$\alpha + \rho + \tau = 1$$

where

α = **absorptivity** (fraction of incident radiant energy absorbed)
ρ = **reflectivity** (fraction of incident radiant energy reflected)
τ = **transmissivity** (fraction of incident radiant energy transmitted)

This is also true for spectral values. For an opaque surface, $\tau = 0$ and $\rho + \alpha = 1$. For a black surface, $\alpha = 1$, $\rho = 0$, and $\tau = 0$.

Kirchhoff's law relates emissivity and absorptivity of any opaque surface from thermodynamic considerations; it states that, for any surface where incident radiation is independent of angle or where the surface emits diffusely, $\varepsilon_\lambda = \alpha_\lambda$. If the surface is gray, or the incident radiation is from a black surface at the same temperature, then $\varepsilon = \alpha$ as well, but many surfaces are not gray. For most surfaces listed in Table 5, the total absorptivity for solar radiation is different from the total emissivity for low-temperature radiation, because ε_λ and α_λ vary with wavelength. Much solar radiation is at short wavelengths. Most emissions from surfaces at moderate temperatures are at longer wavelengths.

Platinum black and gold black are almost perfectly black and have absorptivities of about 98% in the infrared region. A small opening in a large cavity approaches blackbody behavior because most of the incident energy entering the cavity is absorbed by repeated reflection within it, and very little escapes the cavity. Thus, the absorptivity and therefore the emissivity of the opening are close to unity. Some flat black paints also exhibit emissivities of 98% over a wide range of conditions. They provide a much more durable surface than gold or platinum black, and are frequently used on radiation instruments and as standard reference in emissivity or reflectance measurements.

Example 7. In outer space, the solar energy flux on a surface is 1150 W/m². Two surfaces are being considered for an absorber plate to be used on the surface of a spacecraft: one is black, and the other is specially coated for a solar absorptivity of 0.94 and infrared emissivity of 0.1. Coolant flowing through the tubes attached to the plate maintains the plate at 340 K. The plate surface is normal to the solar beam. For each surface, determine the (1) heat transfer rate to the coolant per unit area of the plate, and (2) temperature of the surface when there is no coolant flow.

Solution: For the black surface,

$$\varepsilon = \alpha = 1,\ \rho = 0$$

Absorbed energy flux = 1150 W/m²

At T_s = 340 K, emitted energy flux = $W_b = 5.67 \times 10^{-8} \times 340^4$ = 757.7 W/m².

In space, there is no convection, so an energy balance on the surface gives

Heat flux to coolant = Absorbed energy flux – Emitted energy flux
= 1150 – 757.7 = 392.3 W/m²

For the special surface, use solar absorptivity to determine the absorbed energy flux, and infrared emissivity to calculate the emitted energy flux.

Absorbed energy flux = 0.94 × 1150 = 1081 W/m²
Emitted energy flux = 0.1 × 757.7 = 75.8 W/m²
Heat flux to coolant = 1081 – 75.8 = 1005 W/m²

Table 5 Emissivities and Absorptivities of Some Surfaces

Surface	Total Hemispherical Emissivity	Solar Absorptivity*
Aluminum		
Foil, bright dipped	0.03	0.10
Alloy: 6061	0.04	0.37
Roofing	0.24	
Asphalt	0.88	
Brass		
Oxidized	0.60	
Polished	0.04	
Brick	0.90	
Concrete, rough	0.91	0.60
Copper		
Electroplated	0.03	0.47
Black oxidized in Ebanol C	0.16	0.91
Plate, oxidized	0.76	
Glass		
Polished	0.87 to 0.92	
Pyrex	0.80	
Smooth	0.91	
Granite	0.44	
Gravel	0.30	
Ice	0.96 to 0.97	
Limestone	0.92	
Marble		
Polished or white	0.89 to 0.92	
Smooth	0.56	
Mortar, lime	0.90	
Nickel		
Electroplated	0.03	0.22
Solar absorber, electro-oxidized on copper	0.05 to 0.11	0.85
Paints		
Black		
Parsons optical, silicone high heat, epoxy	0.87 to 0.92	0.94 to 0.97
Gloss	0.90	
Enamel, heated 1000 h at 650 K	0.80	
Silver chromatone	0.24	0.20
White		
Acrylic resin	0.90	0.26
Gloss	0.85	
Epoxy	0.85	0.25
Paper, roofing or white	0.88 to 0.86	
Plaster, rough	0.89	
Refractory	0.90 to 0.94	
Sand	0.75	
Sandstone, red	0.59	
Silver, polished	0.02	
Snow, fresh	0.82	0.13
Soil	0.94	
Water	0.90	0.98
White potassium zirconium silicate	0.87	0.13

Source: Mills (1999)
*Values are for extraterrestrial conditions, except for concrete, snow, and water.

Without coolant flow, heat flux to the coolant is zero. Therefore, absorbed energy flux = emitted energy flux. For the black surface,

$$1150 = 5.67 \times 10^{-8} \times T_s^4 \Rightarrow T_s = 377.1 \text{ K}$$

For the special surface,

$$0.94 \times 1150 = 0.1 \times 5.67 \times 10^{-8} \times T_s^4 \Rightarrow T_s = 660.8 \text{ K}$$

Angle Factor

The foregoing discussion addressed emission from a surface and absorption of radiation leaving surrounding surfaces. Before radiation exchange among a number of surfaces can be addressed, the amount of radiation leaving one surface that is incident on another must be determined.

The fraction of all radiant energy leaving a surface i that is directly incident on surface k is the **angle factor** $\boldsymbol{F_{ik}}$ (also known as **view factor**, **shape factor**, and **configuration factor**). The angle factor from area A_k to area A_j, F_{ki}, is similarly defined, merely by interchanging the roles of i and k. The following relations assume

- All surfaces are gray or black
- Emission and reflection are diffuse (i.e., not a function of direction)
- Properties are uniform over the surfaces
- Absorptivity equals emissivity and is independent of temperature of source of incident radiation
- Material located between radiating surfaces neither emits nor absorbs radiation

These assumptions greatly simplify problems, and give good approximate results in many cases. Some of the relations for the angle factor are given below.

Reciprocity relation.

$$F_{ik} A_i = F_{ki} A_k \tag{24a}$$

Decomposition relation. For three surfaces i, j, and k, with A_{ij} indicating one surface with two parts denoted by A_i and A_j,

$$A_k F_{k\text{-}ij} = A_k F_{k\text{-}i} + A_k F_{k\text{-}j} \tag{24b}$$

$$A_{ij} F_{ij\text{-}k} = A_i F_{i\text{-}k} + A_j F_{j\text{-}k} \tag{24c}$$

Law of corresponding corners. This law is discussed by Love (1968) and Suryanarayana (1995). Its use is shown in Example 8.

Summation rule. For an enclosure with n surfaces, some of which may be inside the enclosure,

$$\sum_{k=1}^{n} F_{ik} = 1 \tag{24d}$$

Note that a concave surface may "see itself," and $F_{ii} \neq 0$ for such a surface.

Numerical values of the angle factor for common geometries are given in Figure 15. For equations to compute angle factors for many configurations, refer to Siegel and Howell (2002).

Example 8. A picture window, 3 m long and 1.8 m high, is installed in a wall as shown in Figure 16. The bottom edge of the window is on the floor, which is 6 by 10 m. Denoting the window by 1 and the floor by 234, find $F_{234\text{-}1}$.

Solution: From decomposition rule,

$$A_{234} F_{234\text{-}1} = A_2 F_{2\text{-}1} + A_3 F_{3\text{-}1} + A_4 F_{4\text{-}1}$$

By symmetry, $A_2 F_{2\text{-}1} = A_4 F_{4\text{-}1}$ and $A_{234\text{-}1} = A_3 F_{3\text{-}1} + 2 A_2 F_{2\text{-}1}$.

$$A_{23} F_{23\text{-}15} = A_2 F_{2\text{-}1} + A_2 F_{2\text{-}5} + A_3 F_{3\text{-}1} + A_3 F_{3\text{-}5}$$

From the law of corresponding corners, $A_2 F_{2\text{-}1} = A_3 F_{3\text{-}5}$, so therefore $A_{23} F_{23\text{-}5} = A_2 F_{2\text{-}5} + A_3 F_{3\text{-}1} + 2 A_2 F_{2\text{-}1}$. Thus,

$$A_{234} F_{234\text{-}1} = A_3 F_{3\text{-}1} + A_{23} F_{23\text{-}15} - A_2 F_{2\text{-}5} - A_3 F_{3\text{-}1} = A_{23} F_{23\text{-}15} - A_2 F_{2\text{-}5}$$

$$A_{234} = 60 \text{ m}^2 \qquad A_{23} = 45 \text{ m}^2 \qquad A_2 = 15 \text{ m}^2$$

From Figure 15A with $Y/X = 10/6 = 1.67$ and $Z/X = 1.8/4.5 = 0.4$, $F_{2315} = 0.061$. With $Y/X = 10/1.5 = 6.66$ and $Z/X = 1.8/1.5 = 1.2$, $F_{25} = 0.041$. Substituting the values, $F_{234\text{-}1} = 1/60(45 \times 0.061 - 15 \times 0.041) = 0.036$.

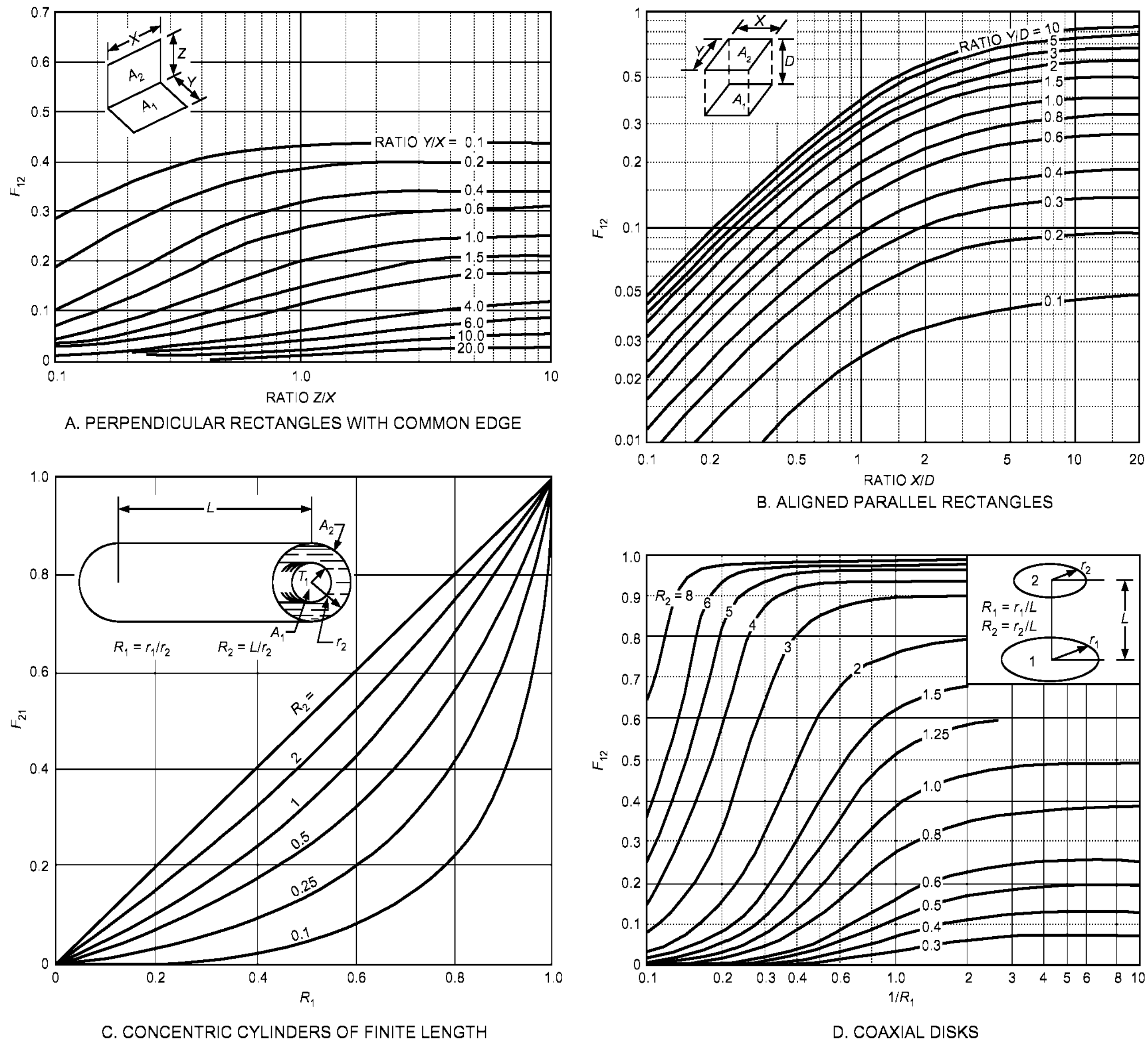


Fig. 15 Radiation Angle Factors for Various Geometries

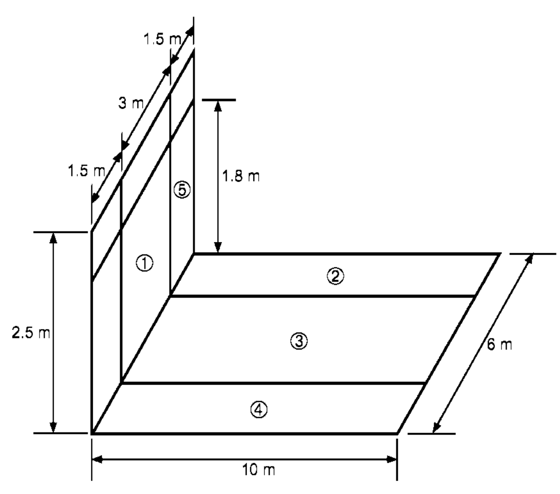


Fig. 16 Diagram for Example 8

Radiant Exchange Between Opaque Surfaces

A surface A_i radiates energy at a rate independent of its surroundings. It absorbs and reflects incident radiation from surrounding surfaces at a rate dependent on its absorptivity. The net heat transfer rate q_i is the difference between the rate radiant energy leaves the surface and the rate of incident radiant energy; it is the rate at which energy must be supplied from an external source to maintain the surface at a constant temperature. The net radiant heat flux from a surface A_i is denoted by q''_i.

Several methods have been developed to solve specific radiant exchange problems. The radiosity method and thermal circuit method are presented here.

Consider the heat transfer rate from a surface of an n-surface enclosure with an intervening medium that does not participate in radiation. All surfaces are assumed gray and opaque. The **radiosity** $\boldsymbol{J_i}$ is the total rate of radiant energy leaving surface i per unit area (i.e., the sum of energy flux emitted and energy flux reflected):

$$J_i = \varepsilon_i W_b + \rho_i G_i \tag{25}$$

where G_i is the total rate of radiant energy incident on surface i per unit area. For opaque gray surfaces, the reflectivity is

$$\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$$

Thus,

$$J_i = \varepsilon_i W_b + (1 - \varepsilon_i) G_i \tag{26}$$

Note that for a black surface, $\varepsilon = 1$, $\rho = 0$, and $J = W_b$.

The net radiant energy transfer q_i is the difference between the total energy leaving the surface and the total incident energy:

$$q_i = A_i (J_i - G_i) \tag{27}$$

Eliminating G_i between Equations (26) and (27),

$$q_i = \frac{W_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} \tag{28}$$

Radiosity Method. Consider an enclosure of n isothermal surfaces with areas of $A_1, A_2, \ldots, A_n$, and emissivities of $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$, respectively. Some may be at uniform but different known temperatures, and the remaining surfaces have uniform but different and known heat fluxes. The radiant energy flux incident on a surface G_i is the sum of the radiant energy reaching it from each of the n surfaces:

$$G_i A_i = \sum_{k=1}^{n} F_{ki} J_k A_k = \sum_{k=1}^{n} F_{ik} J_k A_i \quad \text{or} \quad G_i = \sum_{k=1}^{n} F_{ik} J_k \tag{29}$$

Substituting Equation (29) into Equation (26),

$$J_i = \varepsilon_i W_{bi} + (1 - \varepsilon_i) \sum_{k=1}^{n} F_{ik} J_k \tag{30}$$

Combining Equations (30) and (28),

$$J_i = \frac{q_i}{A_i} + \sum_{k=1}^{n} F_{ik} J_k \tag{31}$$

Note that in Equations (30) and (31), the summation includes surface i.

Equation (30) is for surfaces with known temperatures, and Equation (31) for those with known heat fluxes. An opening in the enclosure is treated as a black surface at the temperature of the surroundings. The resulting set of simultaneous, linear equations can be solved for the unknown J_is.

Once the radiosities (J_is) are known, the net radiant energy transfer to or from each surface or the emissive power, whichever is unknown is determined.

For surfaces where E_{bi} is known and q_i is to be determined, use Equation (28) for a nonblack surface. For a black surface, $J_i = W_{bi}$ and Equation (31) can be rearranged to give

$$\frac{q_i}{A_i} = W_{bi} - \sum_{k=1}^{n} F_{ik} J_k \tag{32}$$

At surfaces where q_i is known and E_{bi} is to be determined, rearrange Equation (28):

$$E_{bi} = J_i + q_i \left(\frac{1 - \varepsilon_i}{A_i \varepsilon_i} \right) \tag{33}$$

The temperature of the surface is then

$$T_i = \left(\frac{W_{bi}}{\sigma} \right)^{1/4} \tag{34}$$

A surface in radiant balance is one for which radiant emission is balanced by radiant absorption (i.e., heat is neither removed from nor supplied to the surface). These are called **reradiating, insulated**, or **refractory surfaces**. For these surfaces, $q_i = 0$ in Equation (31). After solving for the radiosities, W_{bi} can be found by noting that $q_i = 0$ in Equation (33) gives $W_{bi} = J_i$.

Thermal Circuit Method. Another method to determine the heat transfer rate is using thermal circuits for radiative heat transfer rates. Heat transfer rates from surface i to surface k and surface k to surface i, respectively, are given by

$$q_{i\text{-}k} = A_i F_{i\text{-}k} (J_i - J_k) \qquad \text{and} \qquad q_{k\text{-}i} = A_k F_{ik\text{-}i} (J_k - J_i)$$

Using the reciprocity relation $A_i F_{i\text{-}k} = A_k F_{k\text{-}i}$, the net heat transfer rate from surface i to surface k is

$$q_{ik} = q_{i\text{-}k} - q_{k\text{-}i} = A_i F_{i\text{-}k} (J_i - J_k) = \frac{J_i - J_k}{1/A_i F_{i\text{-}k}} \tag{35}$$

Equations (28) and (35) are analogous to the current in a resistance, with the numerators representing a potential difference and the denominator representing a thermal resistance. This analogy can be used to solve radiative heat transfer rates among surfaces, as illustrated in Example 9.

Using angle factors and radiation properties as defined assumes that the surfaces are diffuse radiators, which is a good assumption for most nonmetals in the infrared region, but poor for highly polished metals. Subdividing the surfaces and considering the variation of radiation properties with angle of incidence improves the approximation but increases the work required for a solution. Also note that radiation properties, such as absorptivity, have significant uncertainties, for which the final solutions should account.

Example 9. Consider a 4 m wide, 5 m long, 2.5 m high room as shown in Figure 17. Heating pipes, embedded in the ceiling (1), keep its temperature at 40°C. The floor (2) is at 30°C, and the side walls (3) are at 18°C. The emissivity of each surface is 0.8. Determine the net radiative heat transfer rate to/from each surface.

Solution: Consider the room as a three-surface enclosure. The corresponding thermal circuit is also shown. The heat transfer rates are found after finding the radiosity of each surface by solving the thermal circuit.

From Figure 15A,

$$F_{1\text{-}2} = F_{2\text{-}1} = 0.376$$

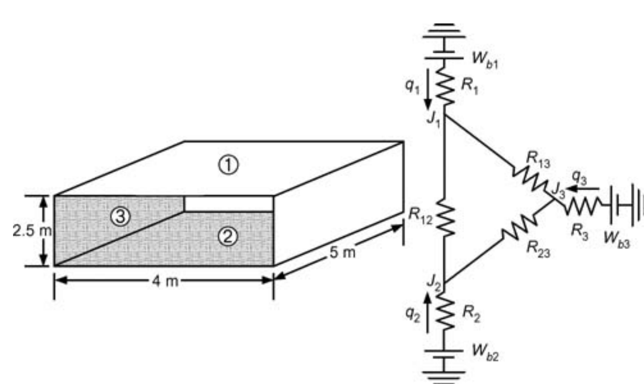


Fig. 17 Diagrams for Example 9

From the summation rule, $F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1$. With $F_{1\text{-}1} = 0$,

$$F_{1\text{-}3} = 1 - F_{1\text{-}2} = 0.624 = F_{2\text{-}3}$$

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.8}{20 \times 0.8} = 0.0125 \text{ m}^{-2} = R_2$$

$$R_3 = \frac{1-\varepsilon_3}{A_3\varepsilon_3} = \frac{1-0.8}{45 \times 0.8} = 0.005\,556 \text{ m}^{-2}$$

$$R_{12} = \frac{1}{A_1F_{1\text{-}2}} = \frac{1}{20 \times 0.376} = 0.133 \text{ m}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{1\text{-}3}} = \frac{1}{20 \times 0.624} = 0.080\,13 \text{ m}^{-2} = R_{23}$$

Performing a balance on each of the three J_i nodes gives

$$\text{Surface 1:} \quad \frac{W_{b1}-J_1}{R_1} + \frac{J_2-J_1}{R_{12}} + \frac{J_3-J_1}{R_{13}} = 0$$

$$\text{Surface 2:} \quad \frac{W_{b2}-J_2}{R_2} + \frac{J_1-J_2}{R_{12}} + \frac{J_3-J_2}{R_{23}} = 0$$

$$\text{Surface 3:} \quad \frac{W_{b3}-J_3}{R_3} + \frac{J_1-J_3}{R_{13}} + \frac{J_2-J_3}{R_{23}} = 0$$

$$W_{b1} = 5.67 \times 10^{-8} \times 313.2^4 = 545.6 \text{ W/m}^2$$

$$W_{b2} = 479.2 \text{ W/m}^2 \qquad W_{b3} = 407.7 \text{ W/m}^2$$

Substituting the values and solving for J_1, J_2, and J_3,

$$J_1 = 524.5 \text{ W/m}^2 \quad J_2 = 475.1 \text{ W/m}^2 \quad J_3 = 418.9 \text{ W/m}^2$$

$$q_1 = \frac{W_{b1}-J_1}{R_1} = \frac{545.6-524.5}{0.0125} = 1688 \text{ W}$$

$$q_2 = 328 \text{ W} \qquad q_3 = -2016 \text{ W}$$

Radiation in Gases

Monatomic and diatomic gases such as oxygen, nitrogen, hydrogen, and helium are essentially transparent to thermal radiation. Their absorption and emission bands are confined mainly to the ultraviolet region of the spectrum. The gaseous vapors of most compounds, however, have absorption bands in the infrared region. Carbon monoxide, carbon dioxide, water vapor, sulfur dioxide, ammonia, acid vapors, and organic vapors absorb and emit significant amounts of energy.

Radiation exchange by opaque solids may be considered a surface phenomenon unless the material is transparent or translucent, though radiant energy does penetrate into the material. However, the penetration depths are small. Penetration into gases is very significant.

Beer's law states that the attenuation of radiant energy in a gas is a function of the product p_gL of the partial pressure of the gas and the path length. The monochromatic absorptivity of a body of gas of thickness L is then

$$\alpha_{\lambda L} = 1 - e^{-\alpha_\lambda L} \tag{36}$$

Because absorption occurs in discrete wavelength bands, the absorptivities of all the absorption bands must be summed over the spectral region corresponding to the temperature of the blackbody radiation passing through the gas. The monochromatic absorption coefficient α_λ is also a function of temperature and pressure of the gas; therefore, detailed treatment of gas radiation is quite complex.

Table 6 Emissivity of CO_2 and Water Vapor in Air at 24°C

Path Length,	CO_2, % by Volume		Relative Humidity, %			
m	0.1	0.3	1.0	10	50	100
3	0.03	0.06	0.09	0.06	0.17	0.22
30	0.09	0.12	0.16	0.22	0.39	0.47
300	0.16	0.19	0.23	0.47	0.64	0.70

Table 7 Emissivity of Moist Air and CO_2 in Typical Room

Relative Humidity, %	ε_g
10	0.10
50	0.19
75	0.22

Estimated emissivity for carbon dioxide and water vapor in air at 24°C is a function of concentration and path length (Table 6). Values are for an isothermal hemispherically shaped body of gas radiating at its surface. Among others, Hottel and Sarofim (1967), Modest (2003), and Siegel and Howell (2002) describe geometrical calculations in their texts on radiation heat transfer. Generally, at low values of p_gL, the mean path length L (or equivalent hemispherical radius for a gas body radiating to its surrounding surfaces) is four times the mean hydraulic radius of the enclosure. A room with a dimensional ratio of 1:1:4 has a mean path length of 0.89 times the shortest dimension when considering radiation to all walls. For a room with a dimensional ratio of 1:2:6, the mean path length for the gas radiating to all surfaces is 1.2 times the shortest dimension. The mean path length for radiation to the 2 by 6 face is 1.18 times the shortest dimension. These values are for cases where the partial pressure of the gas times the mean path length approaches zero ($p_gL \approx 0$). The factor decreases with increasing values of p_gL. For average rooms with approximately 2.4 m ceilings and relative humidity ranging from 10 to 75% at 24°C, the effective path length for carbon dioxide radiation is about 85% of the ceiling height, or 2 m. The effective path length for water vapor is about 93% of the ceiling height, or 2.3 m. The effective emissivity of the water vapor and carbon dioxide radiating to the walls, ceiling, and floor of a room 4.9 by 14.6 m with 2.4 m ceilings is in Table 7.

Radiation heat transfer from the gas to the walls is then

$$q = \sigma A_w \varepsilon_g (T_g^4 - T_w^4) \tag{37}$$

The preceding discussion indicates the importance of gas radiation in environmental heat transfer problems. In large furnaces, gas radiation is the dominant mode of heat transfer, and many additional factors must be considered. Increased pressure broadens the spectral bands, and interaction of different radiating species prohibits simple summation of emissivity factors for the individual species. Nonblackbody conditions require separate calculations of emissivity and absorptivity. Hottel and Sarofim (1967) and McAdams (1954) discuss gas radiation more fully.

THERMAL CONVECTION

Convective heat transfer coefficients introduced previously can be estimated using correlations presented in this section.

Forced Convection

Forced-air coolers and heaters, forced-air- or water-cooled condensers and evaporators, and liquid suction heat exchangers are examples of equipment that transfer heat primarily by forced convection. Although some generalized heat transfer coefficient correlations have been mathematically derived from fundamentals, they are usually obtained from correlations of experimental data. Most correlations for forced convection are of the form

$$\mathrm{Nu} = \frac{hL_c}{k} = f(\mathrm{Re}_{Lc}, \mathrm{Pr})$$

where

Nu = Nusselt number
h = convection heat transfer coefficient
L_c = characteristic length
Re_{Lc} = $\rho V L_c/\mu = V L_c/\nu$
V = fluid velocity
Pr = Prandtl number = $c_p \mu/k$
c_p = fluid specific heat
μ = fluid dynamic viscosity
ρ = fluid density
ν = kinematic viscosity = μ/ρ
k = fluid conductivity

Fluid velocity and characteristic length depend on the geometry.

External Flow. When fluid flows over a flat plate, a **boundary layer** forms adjacent to the plate. The velocity of fluid at the plate surface is zero and increases to its maximum free-stream value at the edge of the boundary layer (Figure 18). Boundary layer formation is important because the temperature change from plate to fluid occurs across this layer. Where the boundary layer is thick, thermal resistance is great and the heat transfer coefficient is small. Flow within the boundary layer immediately downstream from the leading edge is laminar. As flow proceeds along the plate, the laminar boundary layer increases in thickness to a critical value. Then, turbulent eddies develop in the boundary layer, except in a thin laminar sublayer adjacent to the plate.

The boundary layer beyond this point is turbulent. The region between the breakdown of the laminar boundary layer and establishment of the turbulent boundary layer is the **transition region**. Because turbulent eddies greatly enhance heat transport into the main stream, the heat transfer coefficient begins to increase rapidly through the transition region. For a flat plate with a smooth leading edge, the turbulent boundary layer starts at distance x_c from the leading edge where the Reynolds number $\mathrm{Re} = Vx_c/\nu$ is in the range 300 000 to 500 000 (in some cases, higher). In a plate with a blunt front edge or other irregularities, it can start at much smaller Reynolds numbers.

Internal Flow. For tubes, channels, or ducts of small diameter at sufficiently low velocity, the laminar boundary layers on each wall grow until they meet. This happens when the Reynolds number based on tube diameter, $\mathrm{Re} = V_{avg} D/\nu$, is less than 2000 to 2300. Beyond this point, the velocity distribution does not change, and no transition to turbulent flow takes place. This is called **fully developed laminar flow**. When the Reynolds number is greater than 10 000, the boundary layers become turbulent before they meet, and fully developed turbulent flow is established (Figure 19). If flow is turbulent, three different flow regions exist. Immediately next to the wall is a **laminar sublayer**, where heat transfer occurs by thermal conduction; next is a transition region called the **buffer layer**, where both eddy mixing and conduction effects are significant; the final layer, extending to the pipe's axis, is the **turbulent region**, where the dominant mechanism of transfer is eddy mixing.

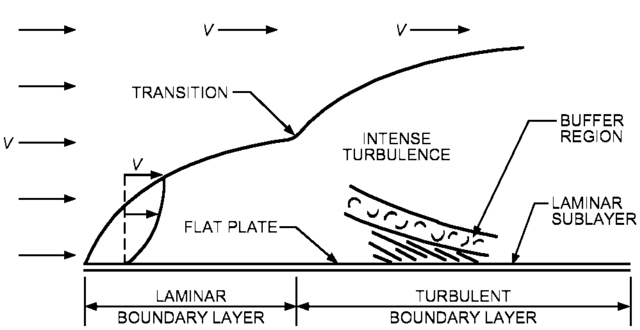


Fig. 18 External Flow Boundary Layer Build-up (Vertical Scale Magnified)

In most equipment, flow is turbulent. For low-velocity flow in small tubes, or highly viscous liquids such as glycol, the flow may be laminar.

The characteristic length for internal flow in pipes and tubes is the inside diameter. For noncircular tubes or ducts, the **hydraulic diameter D_h** is used to compute the Reynolds and Nusselt numbers. It is defined as

$$D_h = 4 \times \frac{\text{Cross-sectional area for flow}}{\text{Total wetted perimeter}} \tag{38}$$

Inserting expressions for cross-sectional area and wetted perimeter of common cross sections shows that the hydraulic diameter is equal to

- The diameter of a round pipe
- Twice the gap between two parallel plates
- The difference in diameters for an annulus
- The length of the side for square tubes or ducts

Table 8 lists various forced-convection correlations. In general, the Nusselt number is determined by the flow geometry, Reynolds number, and Prandtl number. One often useful form for internal flow is known as **Colburn's analogy**:

$$j = \frac{\mathrm{Nu}}{\mathrm{RePr}^{1/3}} = \frac{f_F}{2}$$

where f_F is the Fanning friction factor and j is the Colburn j-factor. It is related to the friction factor by the interrelationship of the transport of momentum and energy in turbulent flow. These factors are plotted in Figure 20.

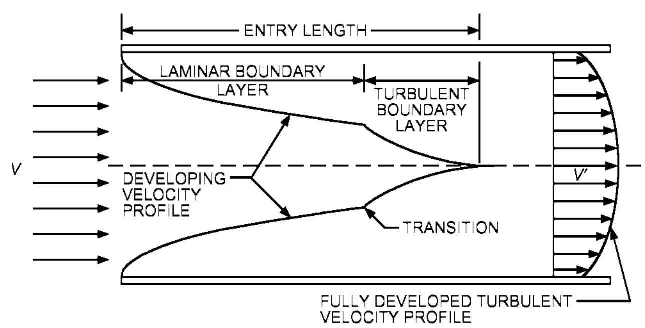


Fig. 19 Boundary Layer Build-up in Entrance Region of Tube or Channel

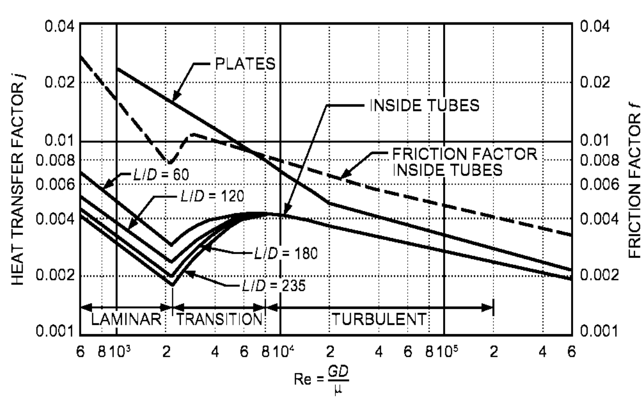
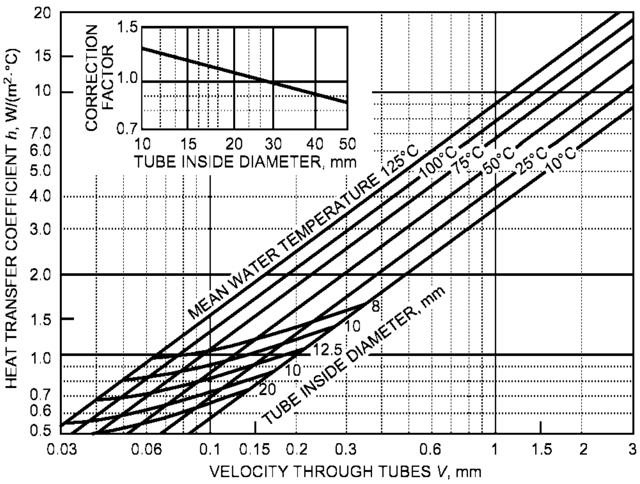


Fig. 20 Typical Dimensionless Representation of Forced-Convection Heat Transfer



Re = 2100 at velocity where diameter curve crosses mean water temperature lines.

Fig. 21 Heat Transfer Coefficient for Turbulent Flow of Water Inside Tubes

Simplified correlations for atmospheric air are also given in Table 8. Figure 21 gives graphical solutions for water.

With a uniform tube surface temperature and heat transfer coefficient, the exit temperature can be calculated using

$$\ln\frac{t_s - t_e}{t_s - t_i} = -\frac{hA}{\dot{m}c_p} \tag{39}$$

where t_i and t_e are the inlet and exit bulk temperatures of the fluid, t_s is the pipe/duct surface temperature, and A is the surface area inside the pipe/duct. The convective heat transfer coefficient varies in the direction of flow because of the temperature dependence of the fluid properties. In such cases, it is common to use an average value of h in Equation (39) computed either as the average of h evaluated at the inlet and exit fluid temperatures or evaluated at the average of the inlet and exit temperatures.

With uniform surface heat flux q'', the temperature of fluid at any section can be found by applying the first law of thermodynamics:

$$\dot{m}\, c_p(t - t_i) = q''A \tag{40}$$

The surface temperature can be found using

$$q'' = h(t_s - t) \tag{41}$$

With uniform surface heat flux, surface temperature increases in the direction of flow along with the fluid.

Natural Convection. Heat transfer with fluid motion resulting solely from temperature differences (i.e., from temperature-dependent density and gravity) is natural (free) convection. Natural-convection heat transfer coefficients for gases are generally much lower than those for forced convection, and it is therefore important not to ignore radiation in calculating the total heat loss or gain. Radiant transfer may be of the same order of magnitude as natural convection, even at room temperatures; therefore, both modes must be considered when computing heat transfer rates from people, furniture, and so on in buildings (see Chapter 9).

Natural convection is important in a variety of heating and refrigeration equipment, such as (1) gravity coils used in high-humidity cold-storage rooms and in roof-mounted refrigerant condensers, (2) the evaporator and condenser of household refrigerators, (3) baseboard radiators and convectors for space heating, and (4) cooling panels for air conditioning. Natural convection is also involved in heat loss or gain to equipment casings and interconnecting ducts and pipes.

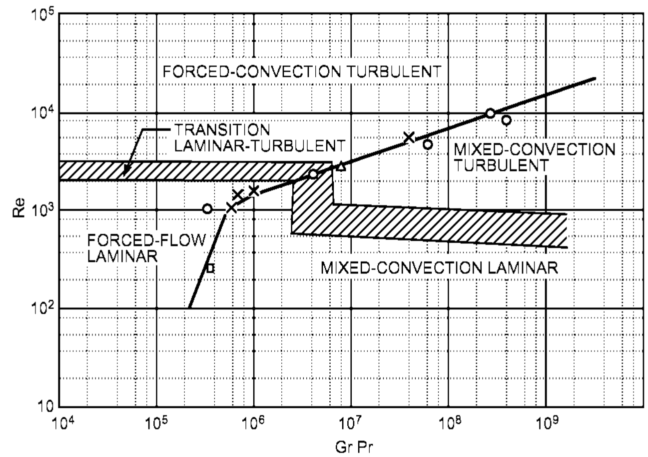


Fig. 22 Regimes of Free, Forced, and Mixed Convection—Flow in Horizontal Tubes

Consider heat transfer by natural convection between a cold fluid and a hot vertical surface. Fluid in immediate contact with the surface is heated by conduction, becomes lighter, and rises because of the difference in density of the adjacent fluid. The fluid's viscosity resists this motion. The heat transfer rate is influenced by fluid properties, temperature difference between the surface at t_s and environment at t_∞, and characteristic dimension L_c. Some generalized heat transfer coefficient correlations have been mathematically derived from fundamentals, but they are usually obtained from correlations of experimental data. Most correlations for natural convection are of the form

$$\mathrm{Nu} = \frac{hL_c}{k} = f(\mathrm{Ra}_{Lc}, \mathrm{Pr})$$

where

- Nu = Nusselt number
- H = convection heat transfer coefficient
- L_c = characteristic length
- K = fluid thermal conductivity
- Ra_{Lc} = Rayleigh number = $g\beta\,\Delta t L_c^3/\nu\alpha$
- Δt = $|t_s - t_\infty|$
- g = gravitational acceleration
- β = coefficient of thermal expansion
- ν = fluid kinematic viscosity = μ/ρ
- α = fluid thermal diffusivity = $k/\rho c_p$
- Pr = Prandtl number = ν/α

Correlations for a number of geometries are given in Table 9. Other information on natural convection is available in the Bibliography under Heat Transfer, General.

Comparison of experimental and numerical results with existing correlations for natural convective heat transfer coefficients indicates that caution should be used when applying coefficients for (isolated) vertical plates to vertical surfaces in enclosed spaces (buildings). Altmayer et al. (1983) and Bauman et al. (1983) developed improved correlations for calculating natural convective heat transfer from vertical surfaces in rooms under certain temperature boundary conditions.

Natural convection can affect the heat transfer coefficient in the presence of weak forced convection. As the forced-convection effect (i.e., the Reynolds number) increases, "mixed convection" (superimposed forced-on-free convection) gives way to pure forced convection. In these cases, consult other sources [e.g., Grigull et al. (1982); Metais and Eckert (1964)] describing combined free and forced convection, because the heat transfer coefficient in the mixed-convection region is often larger than that calculated based on the natural- or forced-convection calculation alone. Metais and Eckert (1964) summarize natural-, mixed-, and forced-convection regimes for vertical

Table 8 Forced-Convection Correlations

I. General Correlation	$\text{Nu} = f(\text{Re}, \text{Pr})$		
II. Internal Flows for Pipes and Ducts: Characteristic length = D, pipe diameter, or D_h, hydraulic diameter.			
$\text{Re} = \dfrac{\rho V_{avg} D_h}{\mu} = \dfrac{\dot m D_h}{A_c \mu} = \dfrac{Q D_h}{A_c \nu} = \dfrac{4\dot m}{\mu P_{wet}} = \dfrac{4Q}{\nu P_{wet}}$	where $\dot m$ = mass flow rate, Q = volume flow rate, P_{wet} = wetted perimeter, A_c = cross-sectional area, and ν = kinematic viscosity (μ/ρ).		
	$\dfrac{\text{Nu}}{\text{Re Pr}^{1/3}} = \dfrac{f}{2}$	Colburn's analogy	(T8.1)
Laminar: Re < 2300	$\text{Nu} = 1.86\left(\dfrac{\text{Re Pr}}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	$\dfrac{L}{D} < \dfrac{\text{Re Pr}}{8}\left(\dfrac{\mu}{\mu_s}\right)^{0.42}$	(T8.2)[a]
Developing	$\text{Nu} = 3.66 + \dfrac{0.065(D/L)\text{Re Pr}}{1 + 0.04[(D/L)\text{Re Pr}]^{2/3}}$		(T8.3)
Fully developed, round	Nu = 3.66	Uniform surface temperature	(T8.4a)
	Nu = 4.36	Uniform heat flux	(T8.4b)
Turbulent:	$\text{Nu} = 0.023\ \text{Re}^{4/5}\text{Pr}^{0.4}$	Heating fluid Re ≥ 10 000	(T8.5a)[b]
Fully developed	$\text{Nu} = 0.023\ \text{Re}^{4/5}\text{Pr}^{0.3}$	Cooling fluid Re ≥ 10 000	(T8.5b)[b]
Evaluate properties at bulk temperature t_b except μ_s and t_s at surface temperature	$\text{Nu} = \dfrac{(f_s/2)(\text{Re} - 1000)\text{Pr}}{1 + 12.7(f_s/2)^{1/2}(\text{Pr}^{2/3} - 1)}\left[1 + \left(\dfrac{D}{L}\right)^{2/3}\right]$ For fully developed flows, set $D/L = 0$.	$f_s = \dfrac{1}{(1.58 \ln \text{Re} - 3.28)^2}$ Multiply Nu by $(T/T_s)^{0.45}$ for gases and by $(\text{Pr}/\text{Pr}_s)^{0.11}$ for liquids	(T8.6)[c]
	$\text{Nu} = 0.027\ \text{Re}^{4/5}\text{Pr}^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	For viscous fluids	(T8.7)[a]
	For noncircular tubes, use hydraulic mean diameter D_h in the equations for Nu for an approximate value of h.		
III. External Flows for Flat Plate: Characteristic length = L = length of plate. Re = VL/ν. All properties at arithmetic mean of surface and fluid temperatures.			
Laminar boundary layer: Re < 5 × 10^5	$\text{Nu} = 0.332\ \text{Re}^{1/2}\text{Pr}^{1/3}$	Local value of h	(T8.8)
	$\text{Nu} = 0.664\ \text{Re}^{1/2}\text{Pr}^{1/3}$	Average value of h	(T8.9)
Turbulent boundary layer: Re > 5 × 10^5	$\text{Nu} = 0.0296\ \text{Re}^{4/5}\text{Pr}^{1/3}$	Local value of h	(T8.10)
Turbulent boundary layer beginning at leading edge: All Re	$\text{Nu} = 0.037\ \text{Re}^{4/5}\text{Pr}^{1/3}$	Average value of h	(T8.11)
Laminar-turbulent boundary layer: Re > 5 × 10^5	$\text{Nu} = (0.37\ \text{Re}^{4/5} - 871)\text{Pr}^{1/3}$	Average value $\text{Re}_c = 5 \times 10^5$	(T8.12)
IV. External Flows for Cross Flow over Cylinder: Characteristic length = D = diameter. Re = VD/ν. All properties at arithmetic mean of surface and fluid temperatures.			
	$\text{Nu} = 0.3 + \dfrac{0.62\ \text{Re}^{1/2}\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\dfrac{\text{Re}}{282\ 000}\right)^{5/8}\right]^{4/5}$	Average value of h	(T8.14)[d]
V. Simplified Approximate Equations: h is in W/(m²·K), V is in m/s, D is in m, and t is in °C.			
Flows in pipes Re > 10 000	Atmospheric air (0 to 200°C): $h = (3.76 - 0.00497t)V^{0.8}/D^{0.2}$		(T8.15a)[e]
	Water (3 to 200°C): $h = (1206 + 23.9t)V^{0.8}/D^{0.2}$		(T8.15b)[e]
	Water (4 to 104°C): $h = (1431 + 20.9t)V^{0.8}/D^{0.2}$ (McAdams 1954)		(T8.15c)[g]
			(T8.15a)
Flow over cylinders	Atmospheric air: 0°C < t < 200°C, where t = arithmetic mean of air and surface temperature.		
	$h = 2.755V^{0.471}/D^{0.529}$	35 < Re < 5000	(T8.16a)
	$h = (4.22 - 0.002\ 57t)V^{0.633}/D^{0.367}$	5000 < Re < 50 000	(T8.16b)
	Water: 5°C < t < 90°C, where t = arithmetic mean of water and surface temperature.		
	$h = (461.8 + 2.01t)V^{0.471}/D^{0.529}$	35 < Re < 5000	(T8.17a)
	$h = (1012 + 9.19t)V^{0.633}/D^{0.367}$	5000 < Re < 50 000	(T8.17b)[f]

Sources: [a]Sieder and Tate (1936), [b]Dittus and Boelter (1930), [c]Gnielinski (1990), [d]Churchill and Bernstein (1977), [e]Based on $\text{Nu} = 0.023\ \text{Re}^{4/5}\text{Pr}^{1/3}$, [f]Based on Morgan (1975). [g]McAdams (1954).

Table 9 Natural Convection Correlations

I. General relationships	$\mathrm{Nu} = f(\mathrm{Ra}, \mathrm{Pr})$ or $f(\mathrm{Ra})$		(T9.1)
Characteristic length depends on geometry	$\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr} \qquad \mathrm{Gr} = \dfrac{g\beta\rho^2\lvert\Delta T\rvert L^3}{\mu^2}$	$\mathrm{Pr} = \dfrac{c_p\mu}{k} \qquad \Delta t = t_s - t_\infty$	
II. Vertical plate			
t_s = constant	$\mathrm{Nu} = 0.68 + \dfrac{0.67\mathrm{Ra}^{1/4}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{4/9}}$	$10^{-1} < \mathrm{Ra} < 10^9$	(T9.2)[a]
Characteristic dimension: L = height Properties at $(t_s + t_\infty)/2$ except β at t_∞	$\mathrm{Nu} = \left\{0.825 + \dfrac{0.387\mathrm{Ra}^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	$10^9 < \mathrm{Ra} < 10^{12}$	(T9.3)[a]
q''_s = constant Characteristic dimension: L = height Properties at $t_{s,\,L/2} - t_\infty$ except β at t_∞ Equations (T9.2) and (T9.3) can be used for vertical cylinders if $D/L > 35/\mathrm{Gr}^{1/4}$ where D is diameter and L is axial length of cylinder	$\mathrm{Nu} = \left\{0.825 + \dfrac{0.387\mathrm{Ra}^{1/6}}{[1+(0.437/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	$10^{-1} < \mathrm{Ra} < 10^{12}$	(T9.4)[a]
III. Horizontal plate			
Characteristic dimension = $L = A/P$, where A is plate area and P is perimeter			
Properties of fluid at $(t_s + t_\infty)/2$			
Downward-facing cooled plate and upward-facing heated plate	$\mathrm{Nu} = 0.96\,\mathrm{Ra}^{1/6}$	$1 < \mathrm{Ra} < 200$	(T9.5)[b]
	$\mathrm{Nu} = 0.59\,\mathrm{Ra}^{1/4}$	$200 < \mathrm{Ra} < 10^4$	(T9.6)[b]
	$\mathrm{Nu} = 0.54\,\mathrm{Ra}^{1/4}$	$2.2 \times 10^4 < \mathrm{Ra} < 8 \times 10^6$	(T9.7)[b]
	$\mathrm{Nu} = 0.15\,\mathrm{Ra}^{1/3}$	$8 \times 10^6 < \mathrm{Ra} < 1.5 \times 10^9$	(T9.8)[b]
Downward-facing heated plate and upward-facing cooled plate	$\mathrm{Nu} = 0.27\,\mathrm{Ra}^{1/4}$	$10^5 < \mathrm{Ra} < 10^{10}$	(T9.9)[b]
IV. Horizontal cylinder			
Characteristic length = d = diameter Properties of fluid at $(t_s + t_\infty)/2$ except β at t_∞	$\mathrm{Nu} = \left\{0.6 + \dfrac{0.387\ \mathrm{Ra}^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	$10^9 < \mathrm{Ra} < 10^{13}$	(T9.10)[c]
V. Sphere			
Characteristic length = D = diameter Properties at $(t_s + t_\infty)/2$ except β at t_∞	$\mathrm{Nu} = 2 + \dfrac{0.589\ \mathrm{Ra}^{1/4}}{[1+(0.469/\mathrm{Pr})^{9/16}]^{4/9}}$	$\mathrm{Ra} < 10^{11}$	(T9.11)[d]
VI. Horizontal wire			
Characteristic dimension = D = diameter Properties at $(t_s + t_\infty)/2$	$\dfrac{2}{\mathrm{Nu}} = \ln\left(1 + \dfrac{3.3}{c\mathrm{Ra}^n}\right)$	$10^{-8} < \mathrm{Ra} < 10^6$	(T9.12)[e]
VII. Vertical wire			
Characteristic dimension = D = diameter; L = length of wire	$\mathrm{Nu} = c(\mathrm{Ra}\,D/L)^{0.25} + 0.763\,c^{(1/6)}(\mathrm{Ra}\,D/L)^{(1/24)}$	$c(\mathrm{Ra}\,D/L)^{0.25} > 2 \times 10^{-3}$	(T9.13)[e]
Properties at $(t_s + t_\infty)/2$	In both Equations (T9.12) and (T9.13), $c = \dfrac{0.671}{[1+(0.492/\mathrm{Pr})^{(9/16)}]^{(4/9)}}$ and $n = 0.25 + \dfrac{1}{10 + 5(\mathrm{Ra})^{0.175}}$		
VIII. Simplified equations with air at mean temperature of 21°C: h is in W/(m²·K), L and D are in m, and Δt is in °C.			
Vertical surface	$h = 1.33\left(\dfrac{\Delta t}{L}\right)^{1/4}$	$10^5 < \mathrm{Ra} < 10^9$	(T9.14)
	$h = 1.26(\Delta t)^{1/3}$	$\mathrm{Ra} > 10^9$	(T9.15)
Horizontal cylinder	$h = 1.04\left(\dfrac{\Delta T}{D}\right)^{1/4}$	$10^5 < \mathrm{Ra} < 10^9$	(T9.16)
	$h = 1.23(\Delta t)^{1/3}$	$\mathrm{Ra} > 10^9$	(T9.17)

Sources: [a]Churchill and Chu (1975a), [b]Lloyd and Moran (1974), Goldstein et al. (1973), [c]Churchill and Chu (1975b), [d]Churchill (1990), [e]Fujii et al. (1986).

and horizontal tubes. Figure 22 shows the approximate limits for horizontal tubes. Other studies are described by Grigull et al. (1982).

Example 10. Chilled water at 5°C flows inside a freely suspended 20 mm OD pipe at a velocity of 2.5 m/s. Surrounding air is at 30°C, 70% rh. The pipe is to be insulated with cellular glass having a thermal conductivity of 0.045 W/(m·K). Determine the radial thickness of the insulation to prevent condensation of water on the outer surface.

Solution: In Figure 23,

$t_{fi} = 5°\mathrm{C} \qquad t_{fo} = 30°\mathrm{C} \qquad d_i = \text{OD of tube} = 0.02\ \mathrm{m}$

k_i = thermal conductivity of insulation material = 0.045 W/(m·K)

From the problem statement, the outer surface temperature t_o of the insulation should not be less than the dew-point temperature of air. The dew-point temperature of air at 30°C, 70% rh = 23.93°C. To determine the outer diameter of the insulation, equate the heat transfer rate per unit length of pipe (from the outer surface of the pipe to the water) to the heat transfer rate per unit length from the air to the outer surface:

Next Page

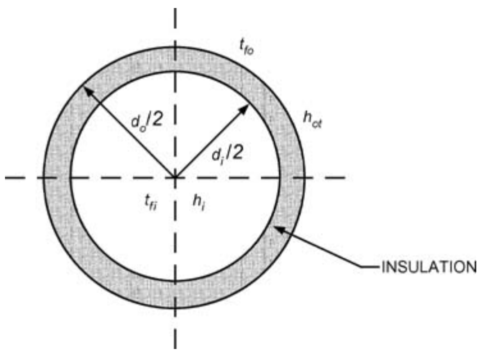


Fig. 23 Diagram for Example 10

$$\frac{t_o - t_{fi}}{\dfrac{1}{h_i d_i} + \dfrac{1}{2k_i}\ln\dfrac{d_o}{d_i}} = \frac{t_{fo} - t_o}{\dfrac{1}{h_{ot} d_o}} \tag{42}$$

Heat transfer from the outer surface is by natural convection to air, so the surface heat transfer coefficient h_{ot} is the sum of the convective heat transfer coefficient h_o and the radiative heat transfer coefficient h_r. With an assumed emissivity of 0.7 and using Equation (4), h_r = 4.3 W/(m^2·K). To determine the value of d_o, the values of the heat transfer coefficients associated with the inner and outer surfaces (h_i and h_o, respectively) are needed. Compute the value of h_i using Equation (T8.6). Properties of water at an assumed temperature of 5°C are

ρ_w = 1000 kg/m^3 μ_w = 0.001 518 (N·s)/m^2 c_{pw} = 4197 W/(m·K)

k_w = 0.5708 W/(m·K) Pr_w = 11.16 $Re_d = \dfrac{\rho v d}{\mu}$ = 32 944

f_s = 0.023 11 Nu_d = 205.6 h_i = 5869 W/(m^2·K)

To compute h_o using Equation (T9.10), the outer diameter of the insulation material must be found. Determine it by iteration by assuming a value of d_o, computing the value of h_o, and determining the value of d_o from Equation (42). If the assumed and computed values of d_o are close to each other, the correct solution has been obtained. Otherwise, recompute h_o using the newly computed value of d_o and repeat the process.

Assume d_o = 0.05 m. Properties of air at t_f = 27°C and 101.325 kPa are

ρ = 1.176 kg/m^3 k = 0.025 66 W/(m·K) $\mu = 1.858 \times 10^{-5}$ (N·s)/m^2

Pr = 0.729 β = 0.003 299 (at 273.15 + 30 = 293.15 K)

Ra = 71 745 Nu = 7.157 h_o = 3.67 W/(m^2·K)

h_{ot} = 3.67 + 4.3 = 7.97 W/(m^2·K)

From Equation (42), d_o = 0.044 28 m. Now, using the new value of 0.044 28 m for the outer diameter, the new values of h_o and h_{ot} are 3.78 W/(m^2·K) and 8.07 W/(m^2·K), respectively. The updated value of d_o is 0.044 03 m. Repeating the process, the final value of d_o = 0.044 01 m. Thus, an outer diameter of 0.045 m (corresponding to an insulation radial thickness of 12.5 mm) keeps the outer surface temperature at 24.1°C, higher than the dew point. (Another method to find the outer diameter is to iterate on the outer surface temperature for different values of d_o.)

HEAT EXCHANGERS

Mean Temperature Difference Analysis

With heat transfer from one fluid to another (separated by a solid surface) flowing through a heat exchanger, the local temperature difference Δt varies along the flow path. Heat transfer rate may be calculated using

$$q = UA\,\Delta t_m \tag{43}$$

where U is the overall uniform heat transfer coefficient, A is the area associated with the coefficient U, and Δt_m is the appropriate mean temperature difference.

For a parallel or counterflow heat exchanger, the mean temperature difference is given by

$$\Delta t_m = \Delta t_1 - \Delta t_2 / \ln(\Delta t_1/\Delta t_2) \tag{44}$$

where Δt_1 and Δt_2 are temperature differences between the fluids at each end of the heat exchanger; Δt_m is the **logarithmic mean temperature difference (LMTD)**. For the special case of $\Delta t_1 = \Delta t_2$ (possible only with a counterflow heat exchanger with equal capacities), which leads to an indeterminate form of Equation (44), $\Delta t_m = t_1 = \Delta t_2$.

Equation (44) for Δt_m is true only if the overall coefficient and the specific heat of the fluids are constant through the heat exchanger, and no heat losses occur (often well-approximated in practice). Parker et al. (1969) give a procedure for cases with variable overall coefficient U. For heat exchangers other than parallel and counterflow, a correction factor [see Incropera et al. (2007)] is needed for Equation (44) to obtain the correct mean temperature difference.

NTU-Effectiveness (ε) Analysis

Calculations using Equations (43) and (44) for Δt_m are convenient when inlet and outlet temperatures are known for both fluids. Often, however, the temperatures of fluids leaving the exchanger are unknown. To avoid trial-and-error calculations, the **NTU-ε method** uses three dimensionless parameters: effectiveness ε, number of transfer units (NTU), and capacity rate ratio c_r; the mean temperature difference in Equation (44) is not needed.

Heat exchanger effectiveness ε is the ratio of actual heat transfer rate to maximum possible heat transfer rate in a counterflow heat exchanger of infinite surface area with the same mass flow rates and inlet temperatures. The maximum possible heat transfer rate for hot fluid entering at t_{hi} and cold fluid entering at t_{ci} is

$$q_{max} = C_{min}(t_{hi} - t_{ci}) \tag{45}$$

where C_{min} is the smaller of the hot $[C_h = ((\dot{m}c_p)_h]$ and cold $[C_c = (\dot{m}c_p)_c]$ fluid capacity rates, W/K; C_{max} is the larger. The actual heat transfer rate is

$$q = \varepsilon q_{max} \tag{46}$$

or a given exchanger type, heat transfer effectiveness can generally be expressed as a function of the **number of transfer units (NTU)** and the **capacity rate ratio** c_r:

$$\varepsilon = f(\text{NTU}, c_r, \text{Flow arrangement}) \tag{47}$$

where

NTU = UA/C_{min}

$c_r = C_{min}/C_{max}$

Effectiveness is independent of exchanger inlet temperatures. For any exchanger in which c_r is zero (where one fluid undergoing a phase change, as in a condenser or evaporator, has an effective $c_p = \infty$), the effectiveness is

$$\varepsilon = 1 - \exp(-\text{NTU}) \tag{48}$$

The mean temperature difference in Equation (44) is then given by

$$\Delta t_m = \frac{(t_{hi} - t_{ci})\varepsilon}{\text{NTU}} \tag{49}$$

After finding the heat transfer rate q, exit temperatures for constant-density fluids are found from